CS 154
Formal Languages and Computability
Assignment #6 Solutions

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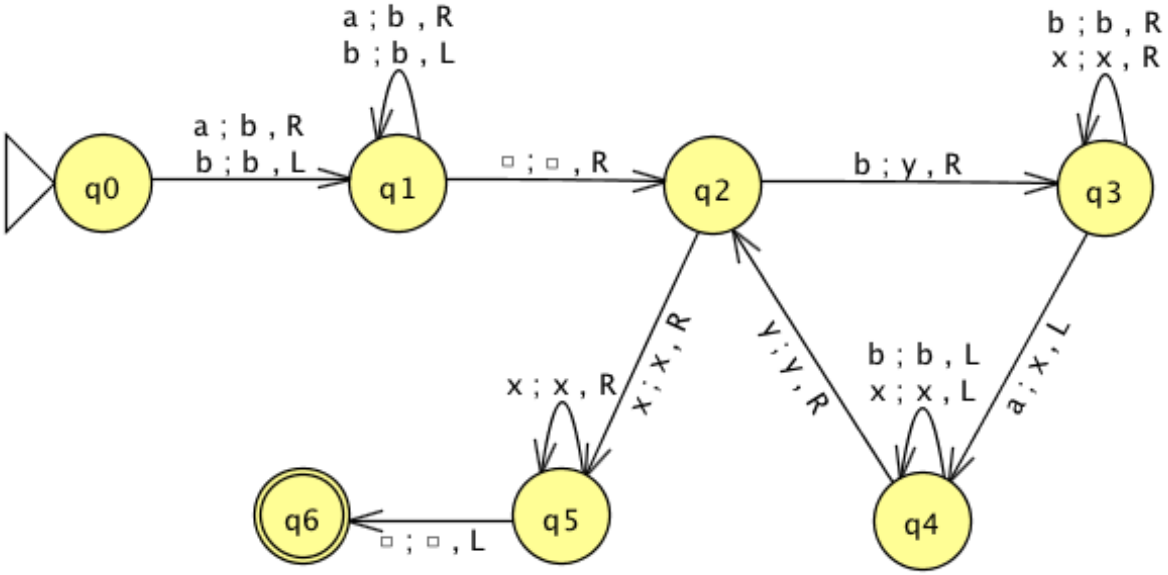
Assignment #6: Problem 1

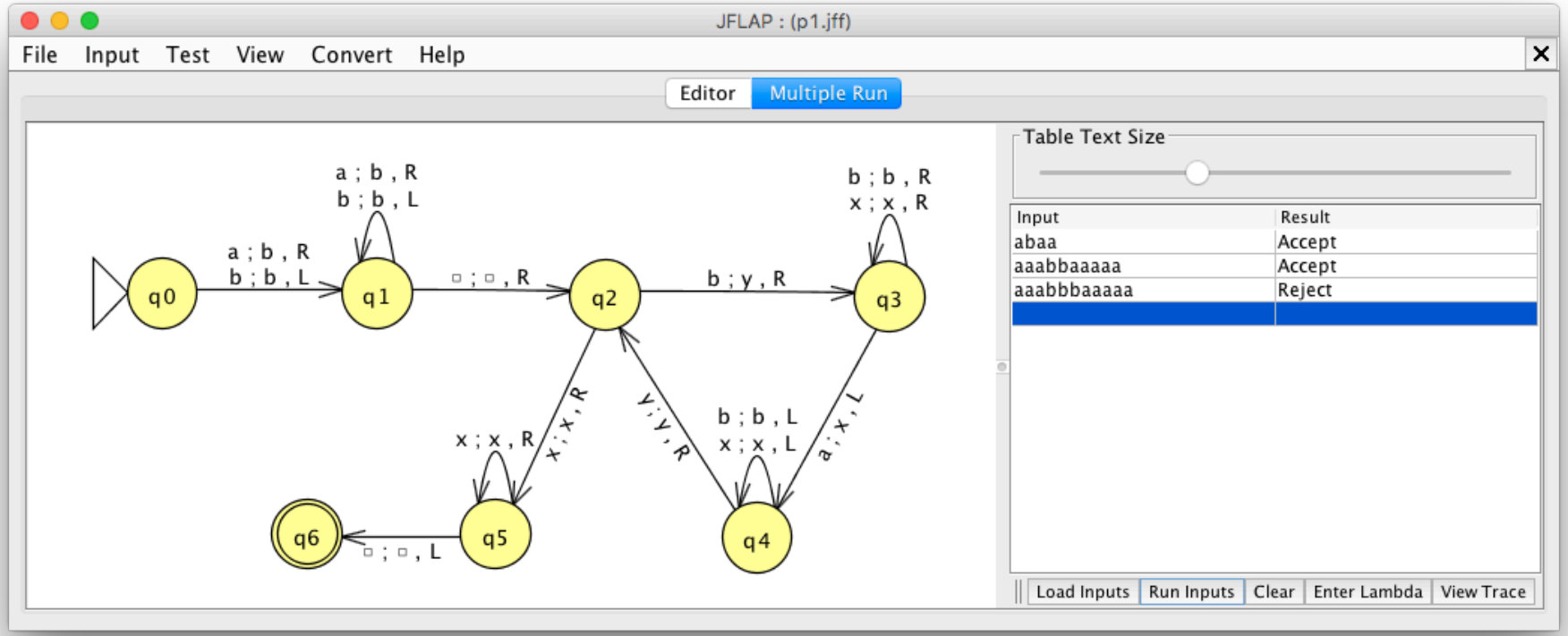
- $L_1 = \{a^n b^m a^{n+m} : n \geq 0, m \geq 1\}$
  - First change all the initial a's to b's.
  - Then verify that the number of b's equals the number of trailing a's by changing each b to an y and each a to an x.



Assignment #6: Problem 1, *cont'd*

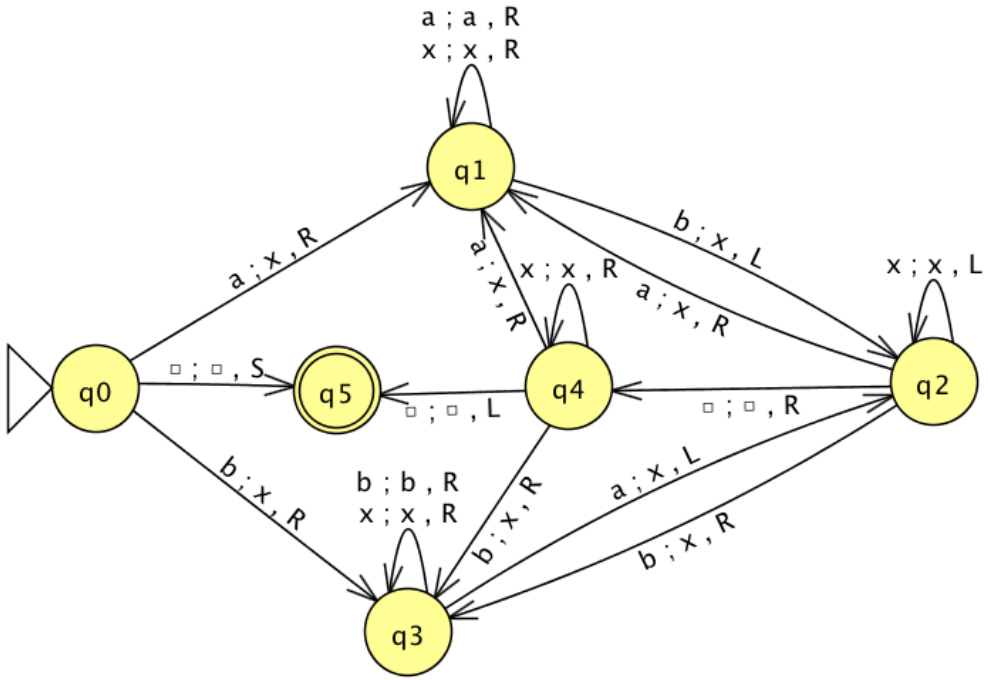
1. q_0 and q_1: Move right to change each initial a to b.
2. q_1: Back left over all the b's.
3. q_2: Change the leftmost b to a y.
4. q_3: Skip right over b's and x's.
   Change the leftmost a to an x and move left.
5. q_4: Skip left over b's and x's. Go to step 3.
6. q_5: Skip right over x's to ensure no more a's.
7. q_6: Accept.

Assignment #6: Problem 1, *cont'd*



Assignment #6: Problem 2

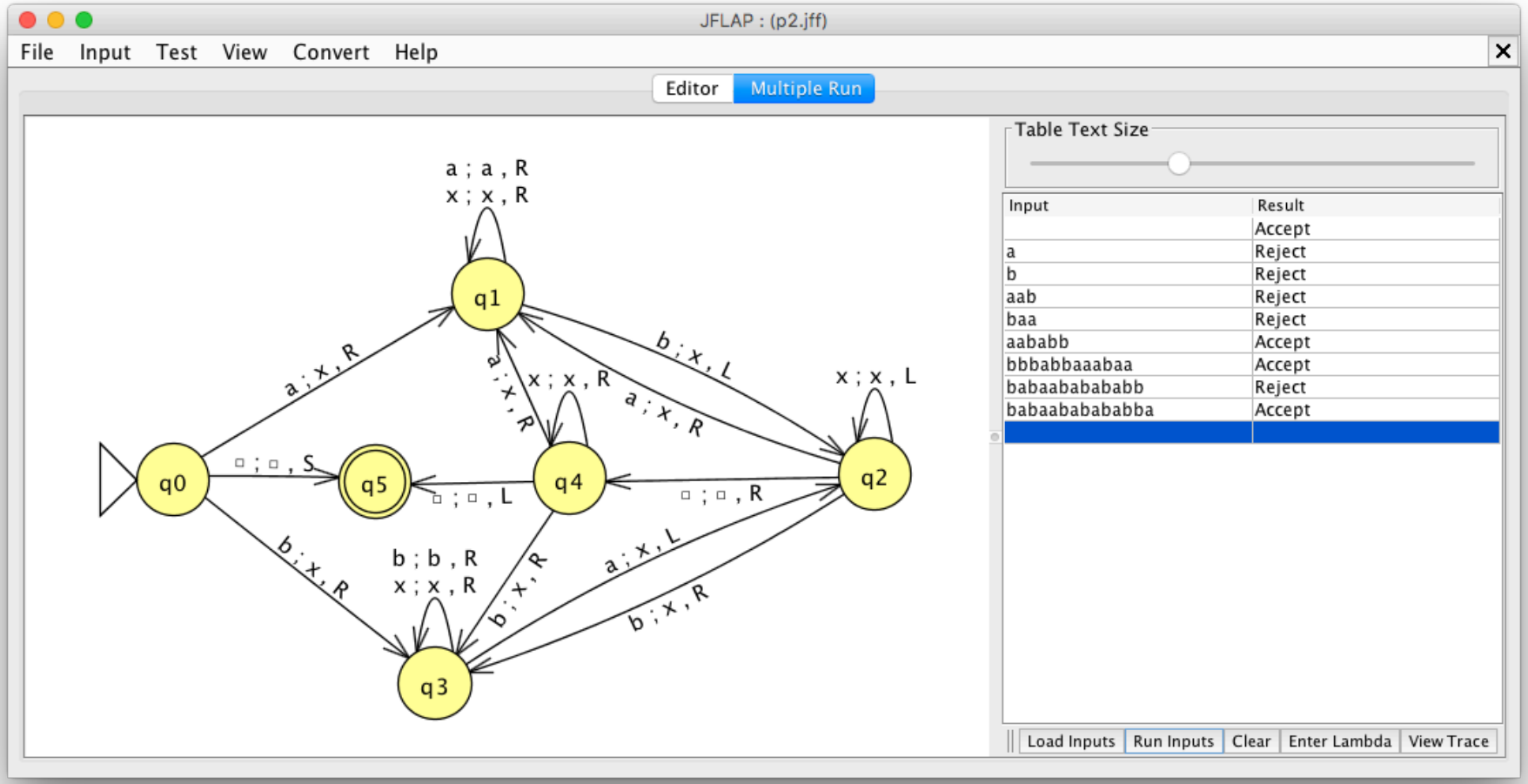
- $L_2 = \{w : n_a(w) = n_b(w)\}$
  - Repeatedly change in pairs an a to an x and a b to an x.
  - If there are no a's or b's left over, then accept.



Assignment #6: Problem 2, *cont'd*

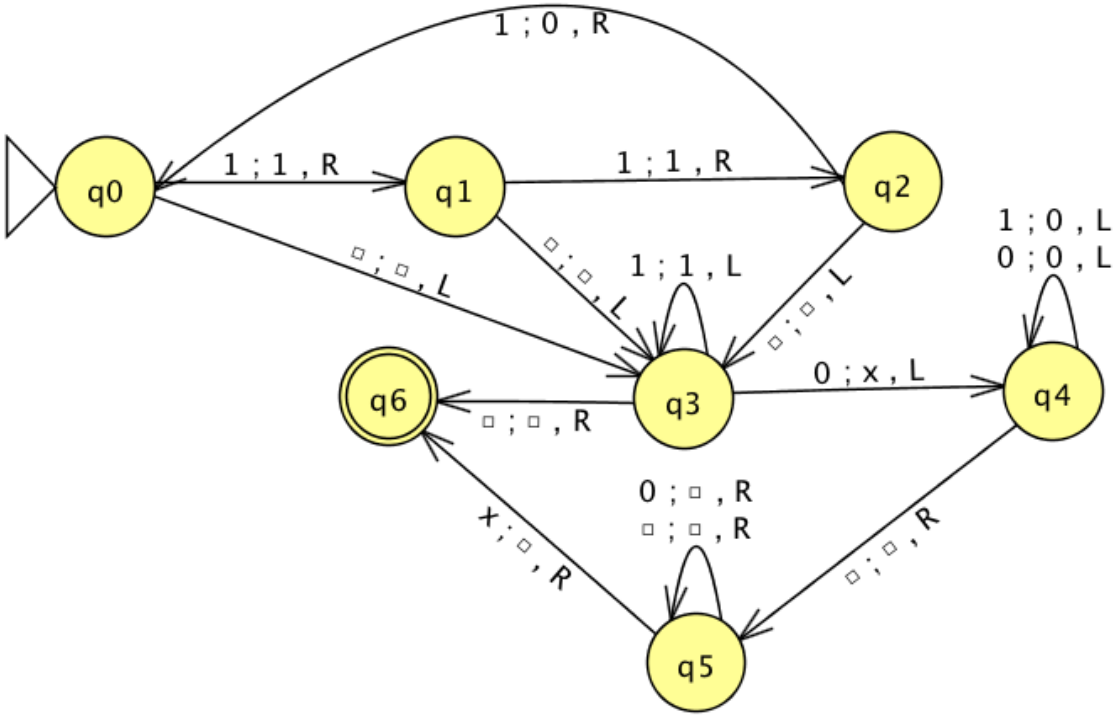
1. q_0: Change the initial a to an x and move right. OR:
   Change the initial b to an x and move right.
2. q_1: Skip right over a's and x's.
   Change the leftmost b to an x and move left.
3. q_3: Skip right over b's and x's.
   Change the leftmost a to an x and move left.
4. q_2: Skip left over x's.
   Change a to x, move right, and go to step 2. OR
   Change b to x, move right, and go to step 3.
5. q_4: Skip right over x's.
   Change a to x, move right, and go to step 2. OR
   Change b to x, move right, and go to step 3.
6. q_5: No a's or b's left, so accept.

Assignment #6: Problem 2, *cont'd*



Assignment #6: Problem 3

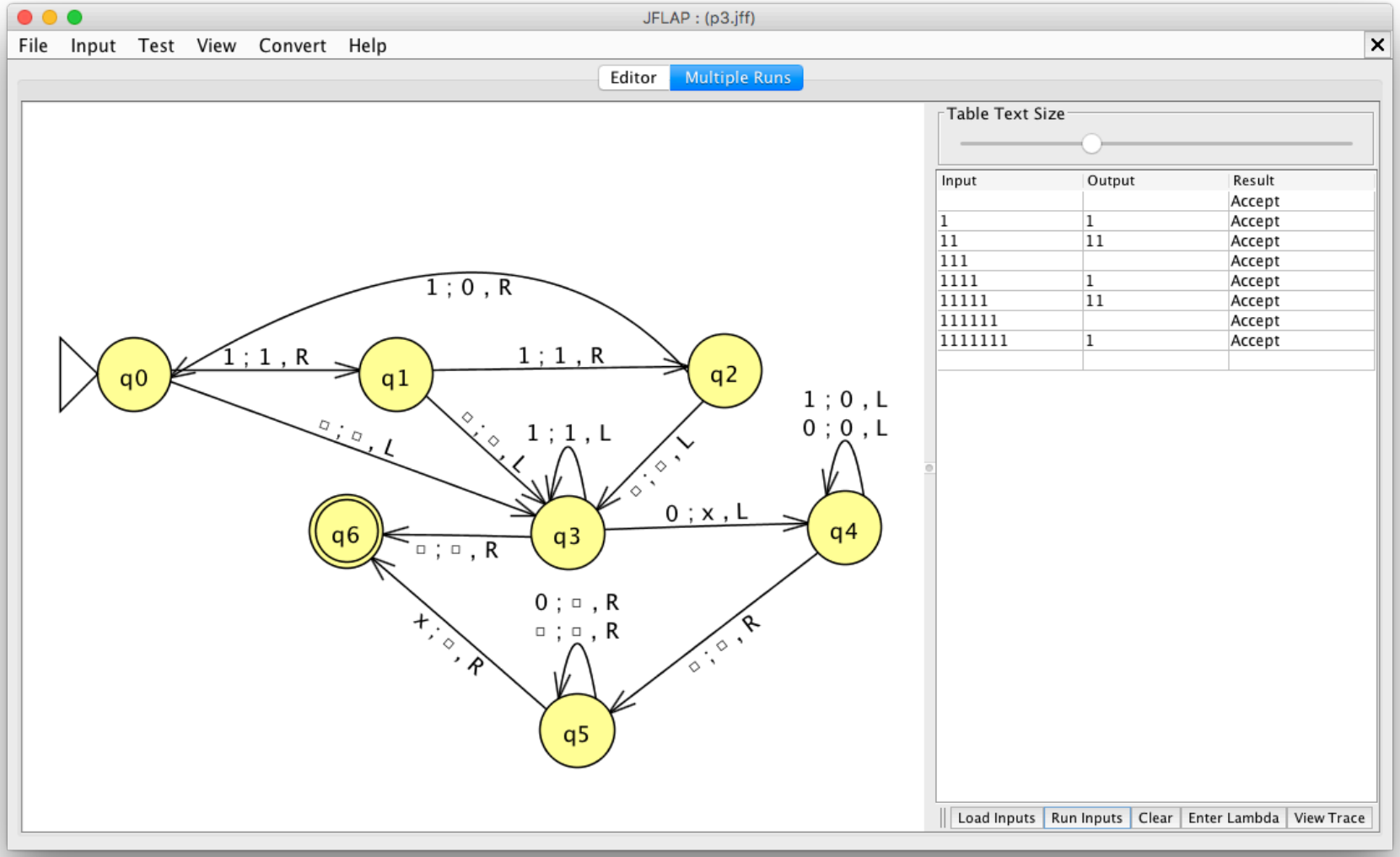
- Compute the function $f(x) = x \bmod 3$
  - Moving left to right, replace every third 1 by a 0.
  - Move left to remove all 1's that are to the left of the 0's.



Assignment #6: Problem 3, *cont'd*

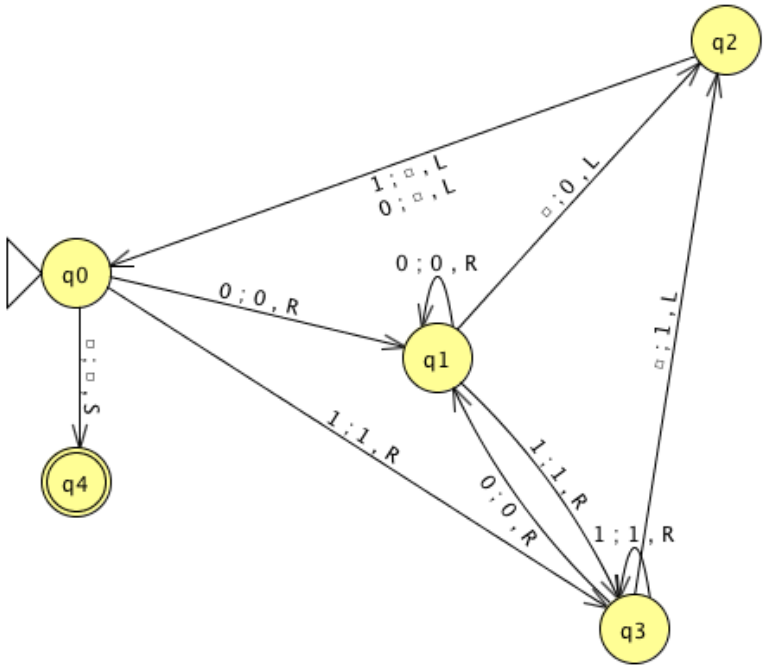
1. q_0 and q_1: Skip right over two consecutive 1's.
2. q_2: Replace every third 1 by a 0 and move right.
3. q_3: Skip left over remaining 1's.
   Change the rightmost 0 to an x and move left.
4. q_4: Move left changing each 1 to a 0.
5. q_5: Move right erasing each 0.
6. q_6: Erase the x and accept.

Assignment #6: Problem 3, *cont'd*



Assignment #6: Problem 4

- Shift an entire input string consisting of 0's and 1's one cell to the right.
  - Shift the string symbols one symbol at a time at the right end, with the head moving left and right.



Assignment #6: Problem 4, *cont'd*

- q_0 , q_1, and q_2: Skip right over 0's and 1's but remember which was the last symbol before a blank.
- q_1: The last symbol was a 0: Change the blank at the right end to a 0 and move left.
- q_2: The last symbol was a 1: Change the blank at the right end to a 1 and move left.
- q_3: Change the copied symbol to a blank and move left. Go to step 1.
- q_4: Accept. The string has shifted right one cell.

Assignment #6: Problem 4 Extra

- Do the right shift without moving the head left.
  - Remember the initial symbol and change it to a blank.
  - Skip to the right runs of 0's and 1's and change the first symbol different from the run to the other symbol.
  - Change the first blank at the right end to the symbol of the last run.

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11100011Δ
Δ1110011Δ
Δ1110001Δ
Δ11100011
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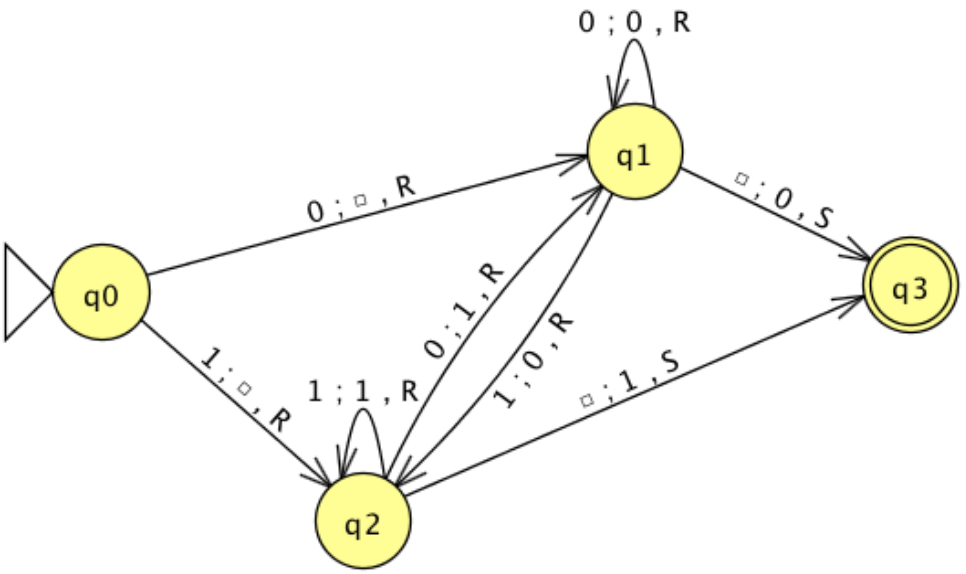

Assignment #6: Problem 4 Extra, *cont'd*

1. q_0 : Change 0 to a blank and move right. OR:
   Change 1 to a blank and move right.
2. q_1: Skip right over a run of consecutive 0's.
   Change a 1 to a 0 and move right.
   Change a blank to a 0.
3. q_2: Skip right over a run of consecutive 1's.
   Change a 0 to a 1 and move right.
   Change a blank to a 1.
4. q_3: Accept. The string has shifted right one cell.