CS 154
Formal Languages and Computability
Assignment #5 Solutions

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Context-Free Pumping Lemma

- If L is an infinite context-free language, then there exists some positive integer m such that for <u>any</u> string w in L with $|w| \geq m$
  - We can decompose $w = uvxyz$
  - with $|vxy| \leq m$
  - and $|vy| \geq 1$ (i.e., vy is nonempty)
  - such that uv^ixy^iz is also in L, with $i = 0, 1, 2, \ldots$
- In other words, strings in a context-free language can be pumped, and the pumped strings are also in the language.

Assignment #5: Problem 1

- Use the pumping lemma to show that the language $L_1 = \{a^n b^{2n} a^n : n \geq 0\}$ is not context-free.

- Assume that L_1 is context free and so the pumping lemma must hold for any string w in L_1 .
- Choose $w = a^m b^{2m} a^m$ which is in L_1.
- Decompose $w = uvxyz$.

We don't know what m equals, only that it exists.

- Since $|vxy| \leq m$, string vy cannot contain symbols from all three blocks a^m, b^{2m}, and a^m.
- Choose $i = 0$ to remove v and y.
- The remaining string uxz cannot be in L_1, a contradiction, and so L_1 is not context-free.

Assignment #5: Problem 2

- Use the pumping lemma to show that the language $L_2 = \{a^n : n \text{ is a perfect square}\}$ is not context-free.

- Assume that L_2 is context-free and so the pumping lemma must hold for any string w in L_2.
- Choose $w = uvxyz = a^{m^2}$ which is in L_2, and $v = a^j$ and $y = a^k$.
- The pumped strings will be $w_i = a^{m^2+(i-1)(j+k)}$ for $i = 0, 1, 2, \ldots$
  - w_0 is the string without v and y.
- But $w_0 = a^{m^2-(j+k)} \notin L_2$ because since $j + k \leq m$, $m^2 - (j + k) \geq m^2 - m = m(m - 1) > (m - 1)^2$ *i.e.*, $|w_0|$ is between two consecutive perfect squares.
- This is a contradiction, so L_2 is not context-free.

Assignment #5: Problem 3

- Use the pumping lemma to show that the language $L_3 = \{a^n : n \text{ is prime}\}$ is not context-free.
- Assume that L_3 is context-free and so the pumping lemma must hold for any string w in L_3.
- Choose the smallest prime n such that $n \geq m$.
- Then $w = uvxyz = a^n$ is in L_3, and $v = a^j$ and $y = a^k$.
- The pumped strings will be $w_i = a^{n+(i-1)(j+k)}$ for $i = 0, 1, 2, \ldots$
- But $w_{n+1} = a^{n+n(j+k)} = a^{n(1+j+k)} \notin L_3$ because $n(1 + j + k)$ is not prime.
  - It's the product of two integers that are each > 1.
- This is a contradiction, so L_3 is not context-free.