CS 154
Formal Languages and Computability
Assignment #2 Solutions

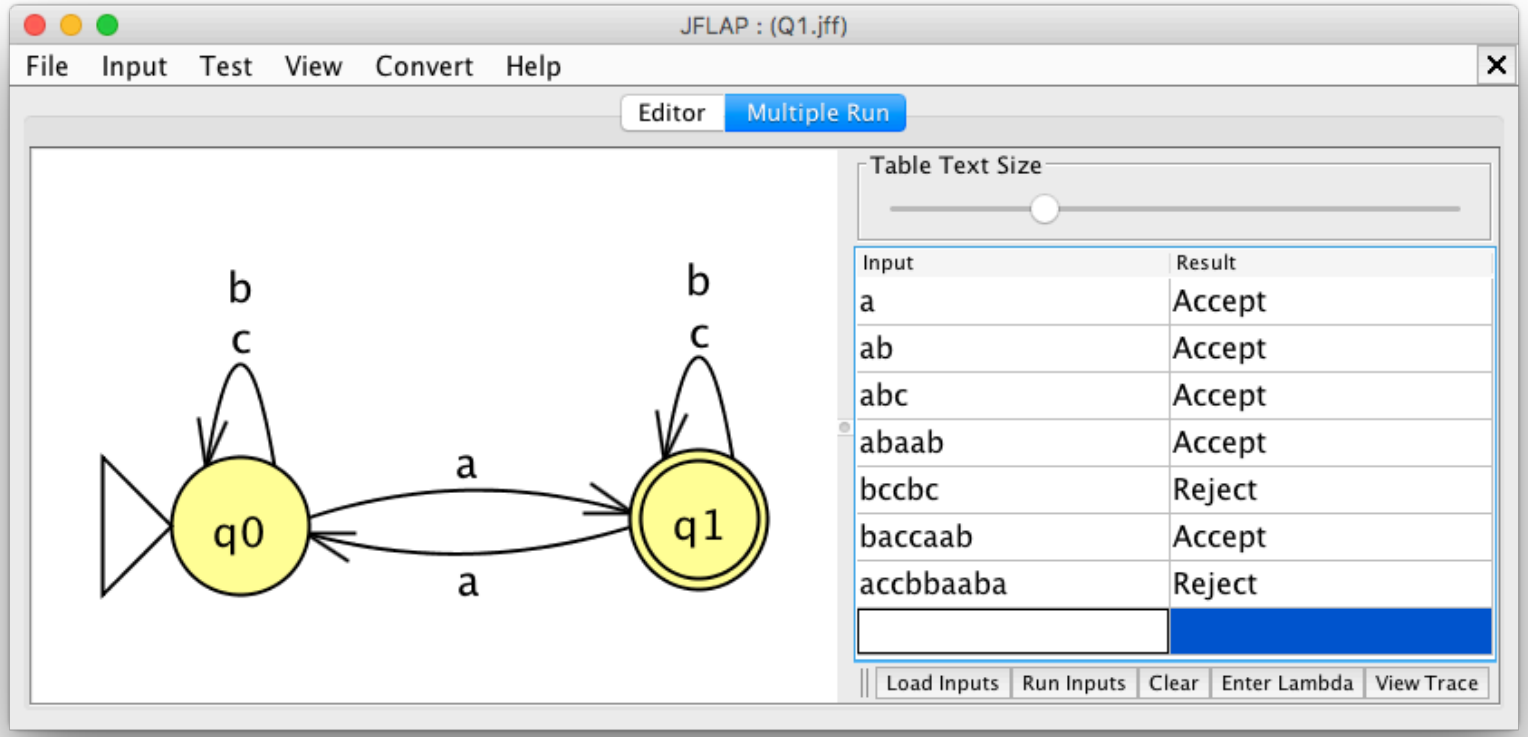
Department of Computer Science
San Jose State University

Spring 2016

Instructor: Ron Mak
www.cs.sjsu.edu/~mak

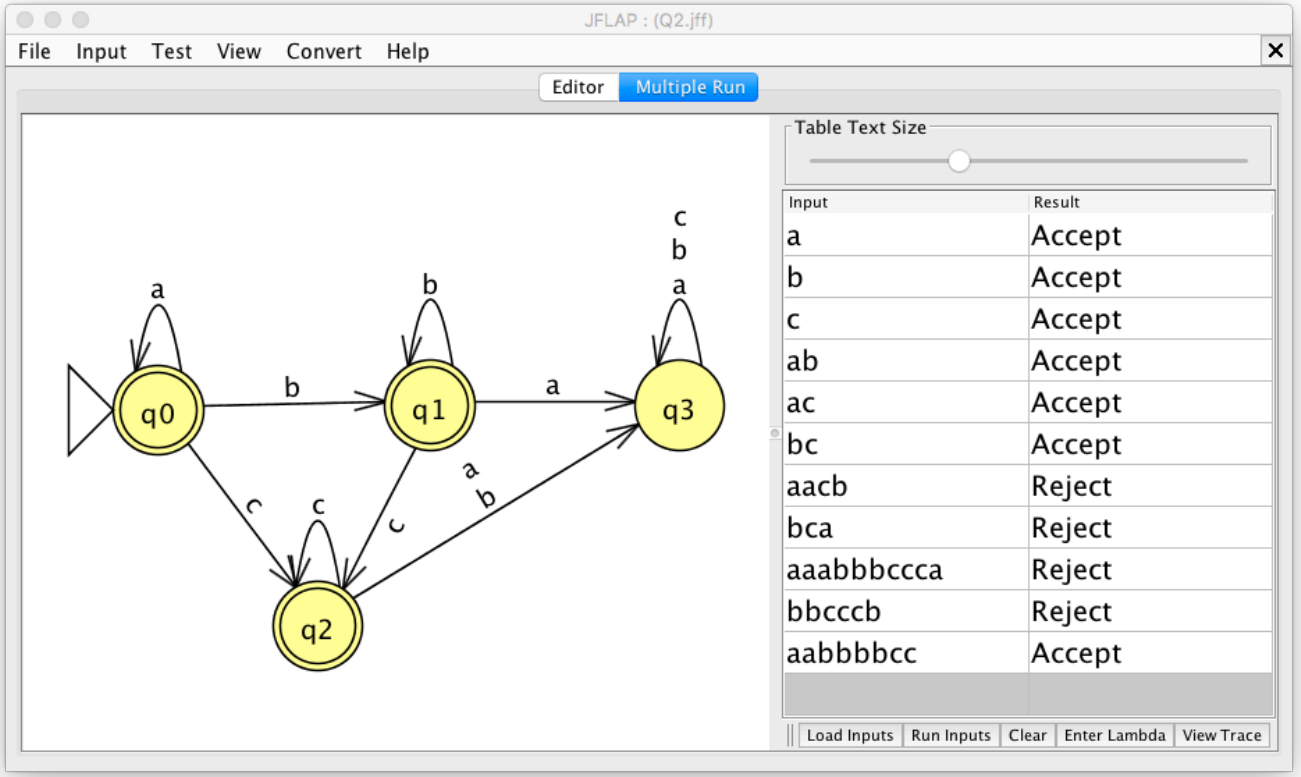
Assignment #2: Question 1

- Use JFLAP to construct the transition graph for the DFA that accepts all strings (and only those strings) on the alphabet $\{a, b, c\}$ that have an odd number of a's.



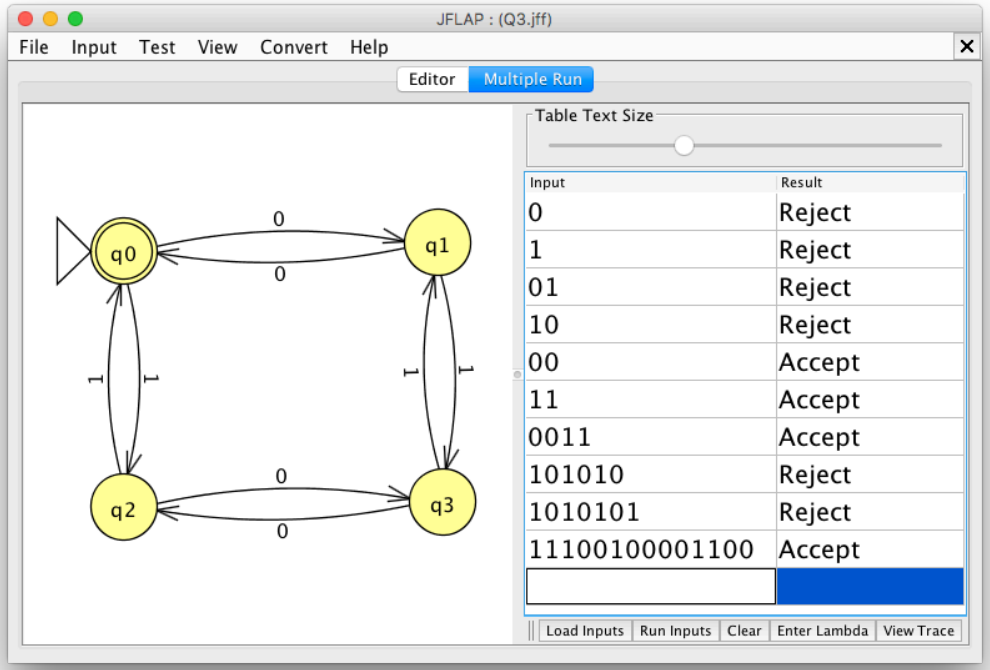
Assignment #2: Question 2

- Use JFLAP to construct the transition graph for the DFA that accepts all strings (and only those strings) on the alphabet $\{a, b, c\}$ that have the symbols in alphabetical order.



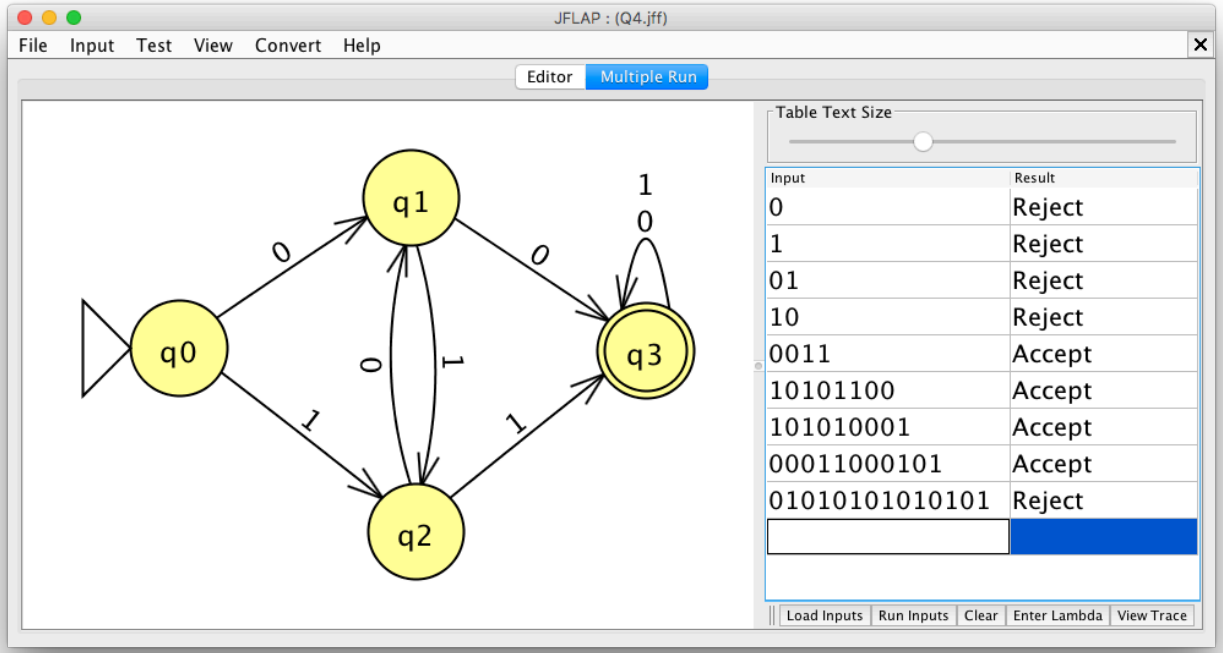
Assignment #2: Question 3

- Describe in words the strings that the following DFA accepts and demonstrate your answer with some sample strings.
  - All strings that contain an even number of 0's or an even number of 1's.



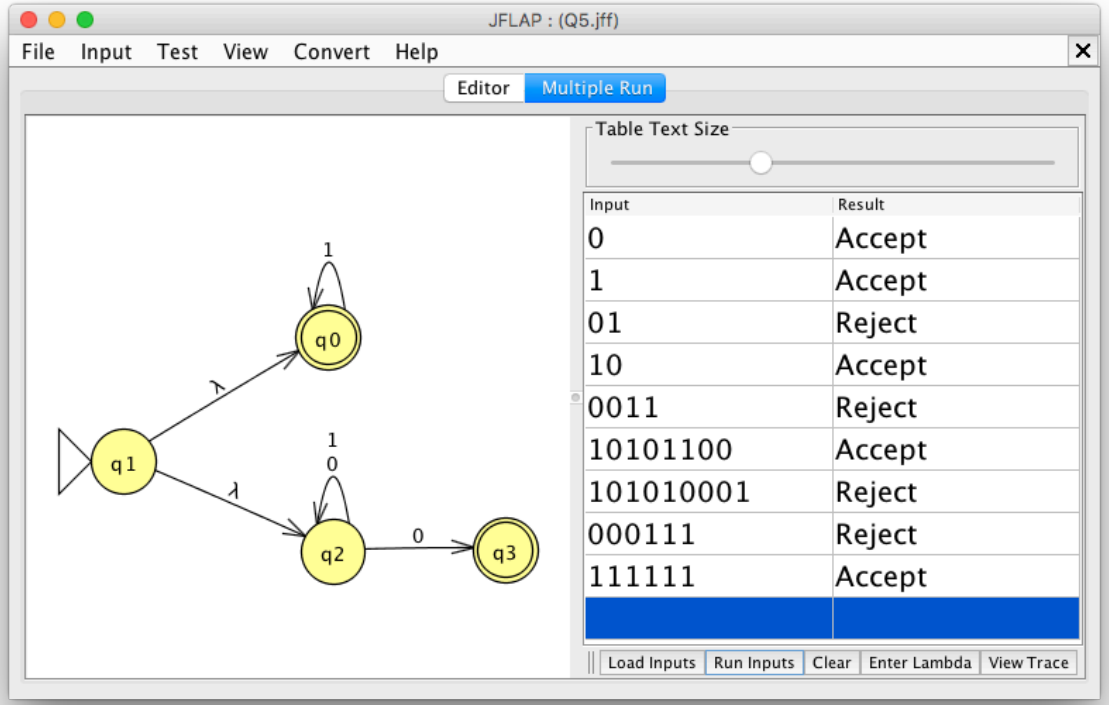
Assignment #2: Question 4

- Describe in words the strings that the following DFA accepts and demonstrate your answer with some sample strings.
  - All strings that contain either two consecutive 0's or two consecutive 1's.

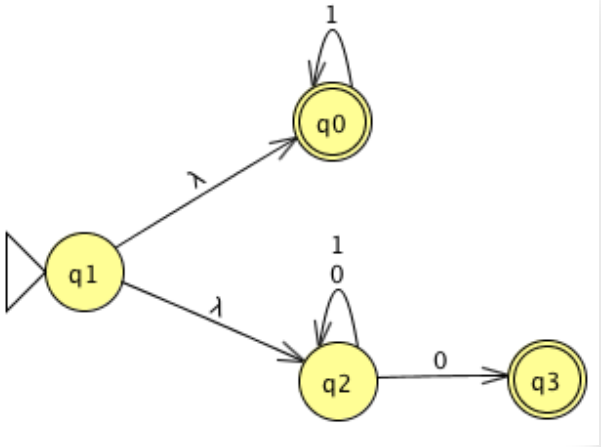


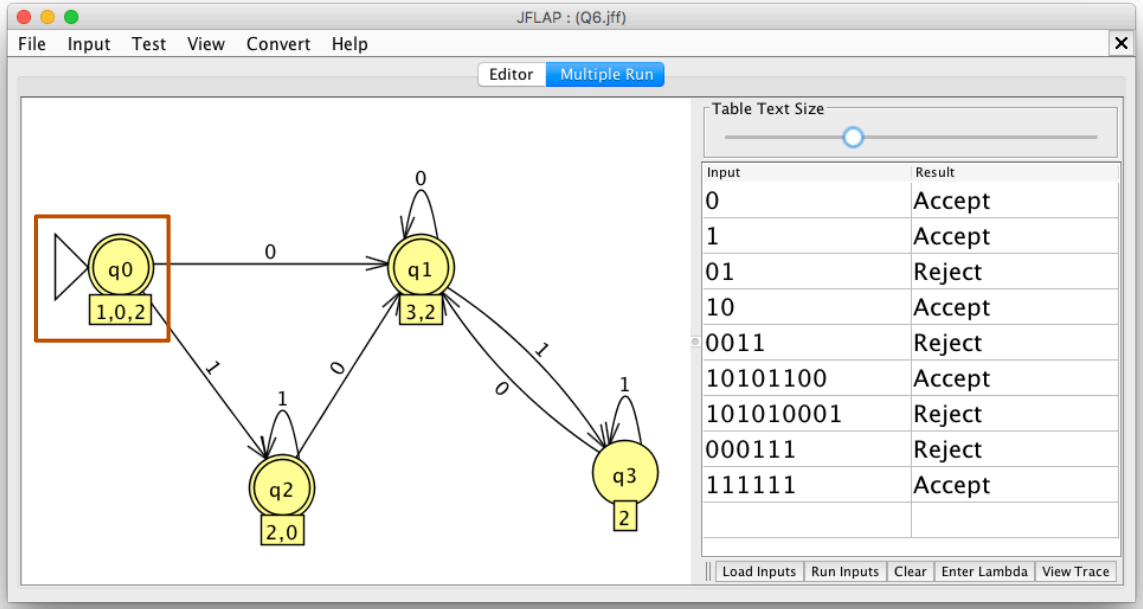
Assignment #2: Question 5

- Describe in words the strings that the following NFA accepts and demonstrate your answer with some sample strings.
  - All strings that contain all 1's or end with a 0.



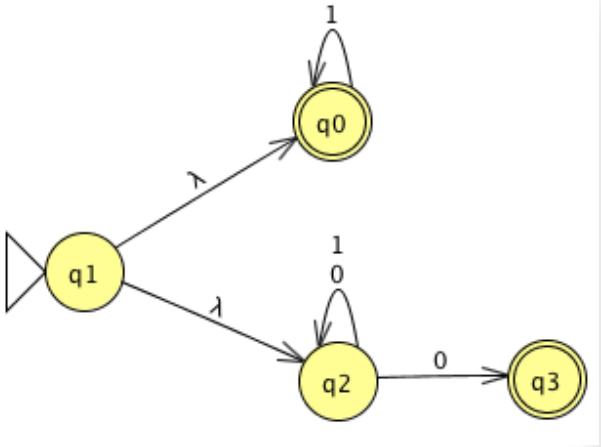
Assignment #2: Question 6

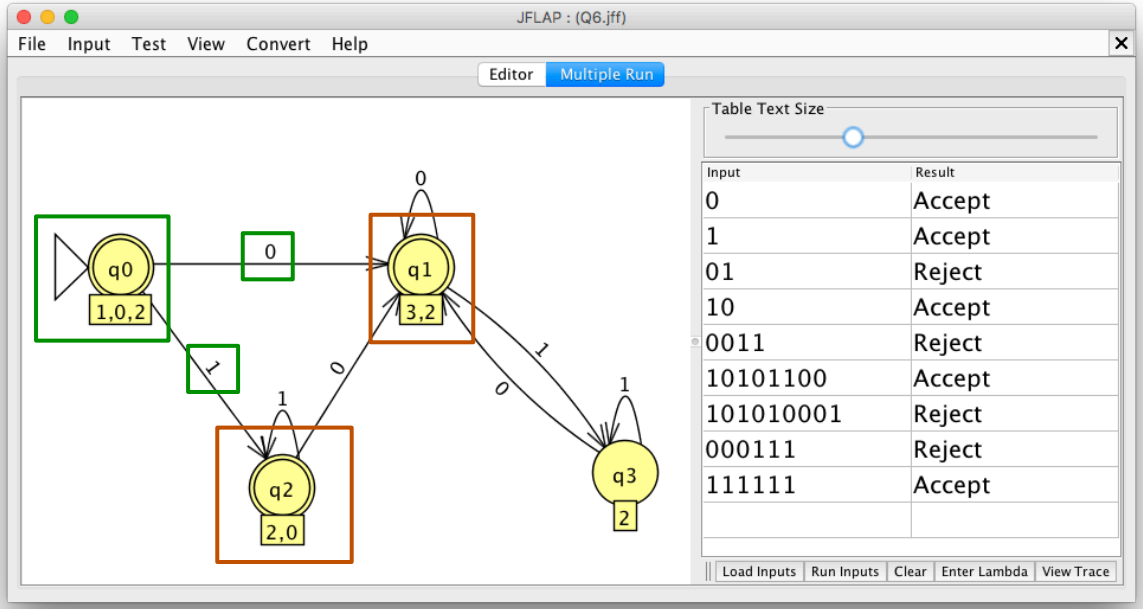




- NFA start state q_0 has λ-transitions to states q_1 and q_2, so label the DFA start state $\{q_0, q_1, q_2\}$.

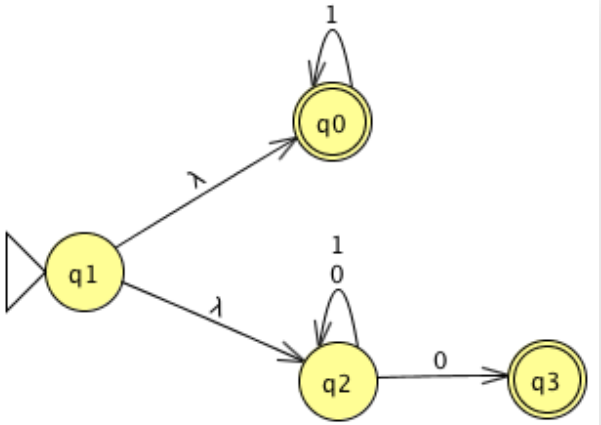
Assignment #2: Question 6, *cont'd*

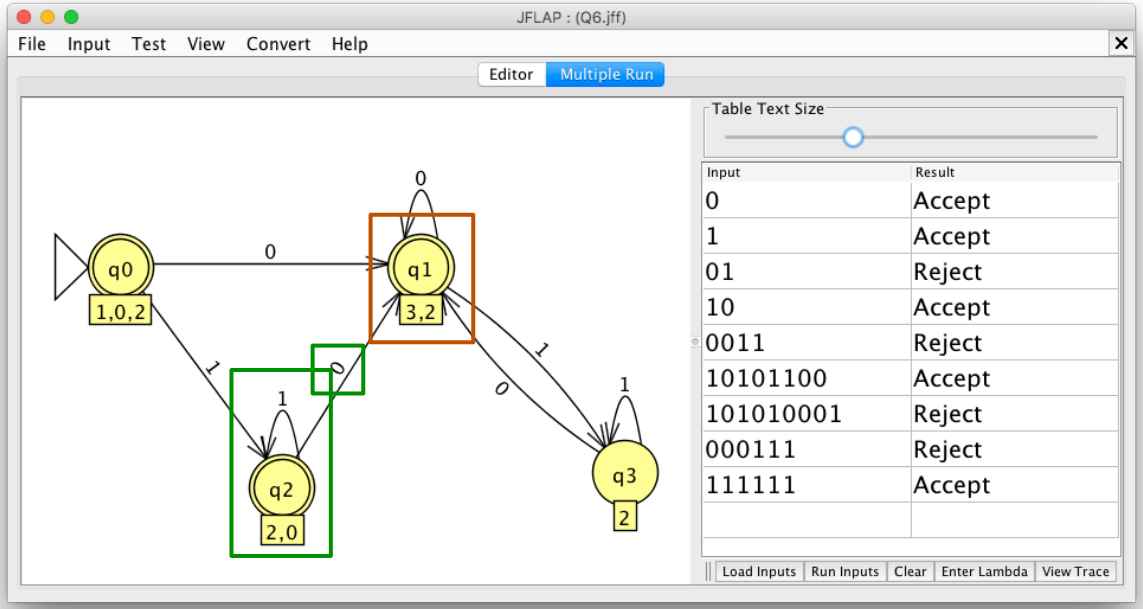




- NFA: $\delta(q_0, 0) = \phi$ and $\delta(q_1, 0) = \phi$ and $\delta(q_2, 0) = \{q_2, q_3\}$, so DFA $\delta(\{q_0, q_1, q_2\}, 0) = \{q_2, q_3\}$. Perform the union.
- NFA: $\delta(q_0, 1) = \{q_0\}$ and $\delta(q_1, 1) = \phi$ and $\delta(q_2, 1) = \{q_2\}$, so DFA $\delta(\{q_0, q_1, q_2\}, 1) = \{q_0, q_2\}$.

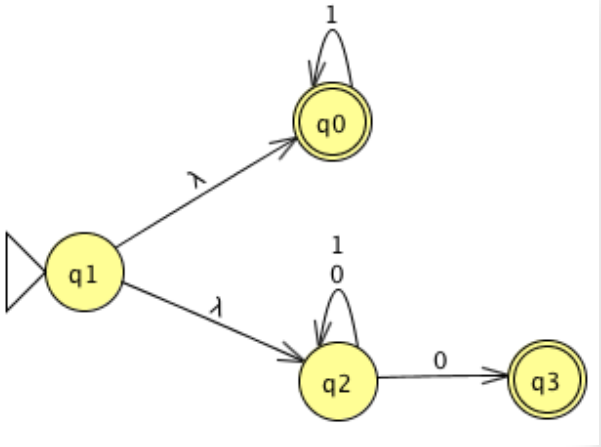
Assignment #2: Question 6, *cont'd*

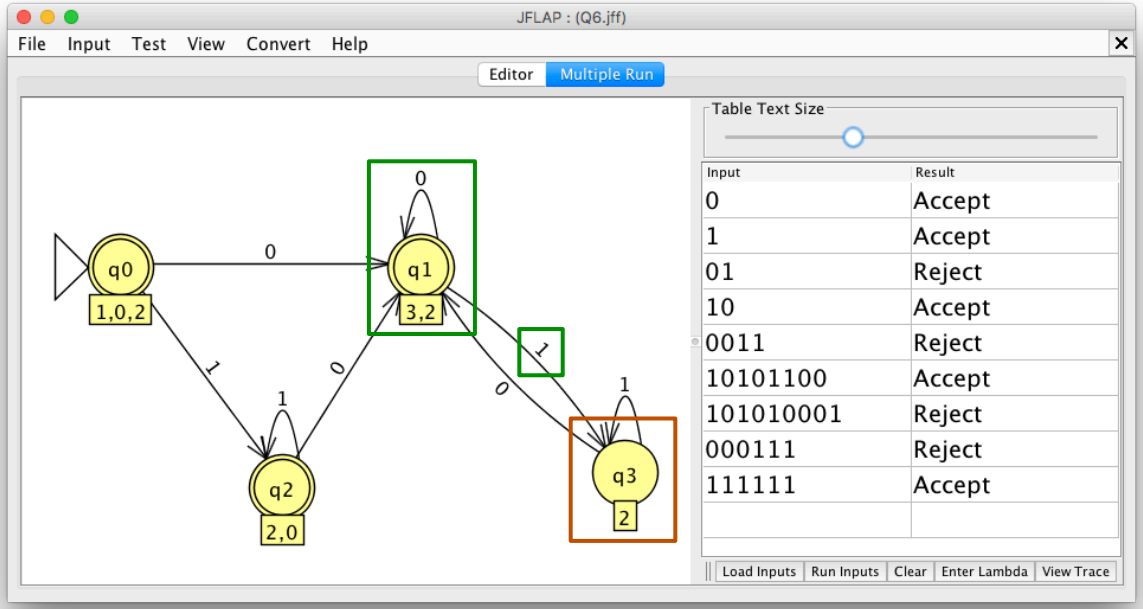




- NFA: $\delta(q_0, 0) = \phi$ and $\delta(q_2, 0) = \{q_2, q_3\}$, so DFA $\delta(\{q_0, q_2\}, 0) = \{q_2, q_3\}$.
- NFA: $\delta(q_0, 1) = \{q_0\}$ and $\delta(q_2, 1) = \{q_2\}$, so DFA $\delta(\{q_0, q_2\}, 1) = \{q_0, q_2\}$.

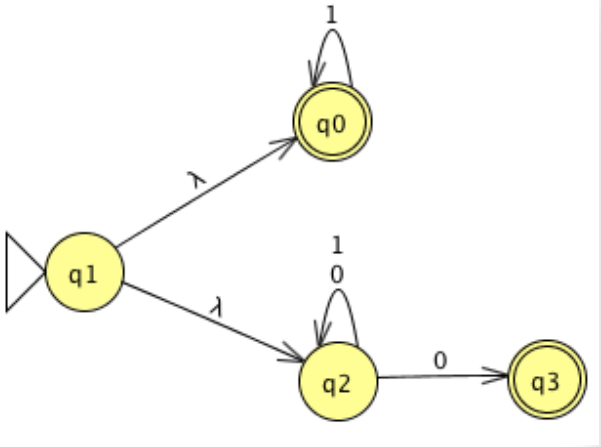
Assignment #2: Question 6, *cont'd*

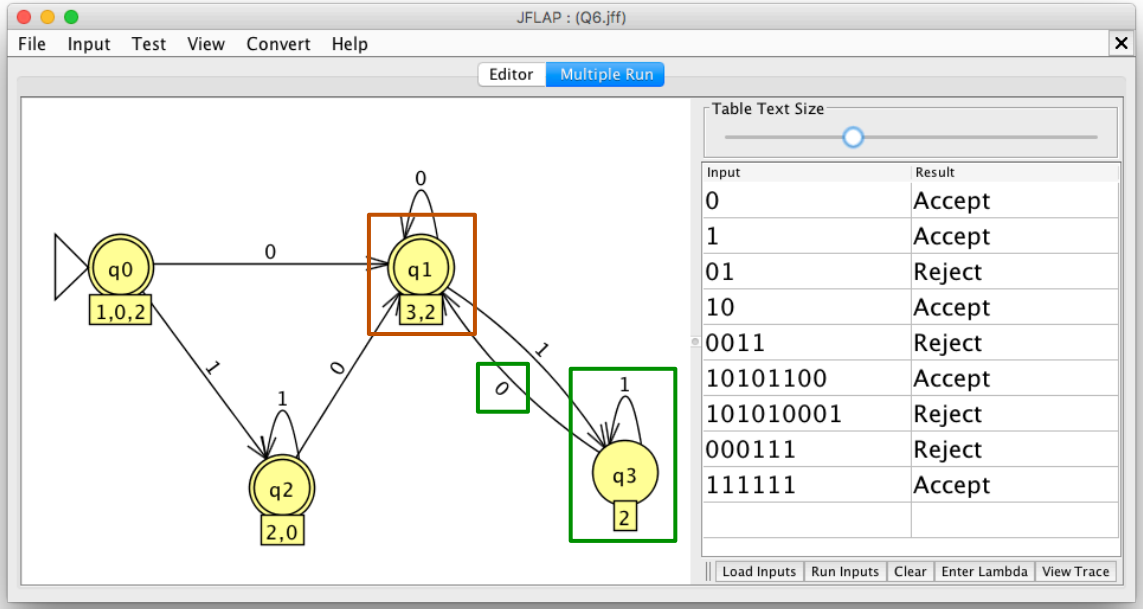




- NFA: $\delta(q_2, 0) = \{q_2, q_3\}$ and $\delta(q_3, 0) = \phi$, so DFA $\delta(\{q_2, q_3\}, 0) = \{q_2, q_3\}$.
- NFA: $\delta(q_2, 1) = \{q_2\}$ and $\delta(q_3, 1) = \phi$, so DFA $\delta(\{q_2, q_3\}, 1) = \{q_2\}$.

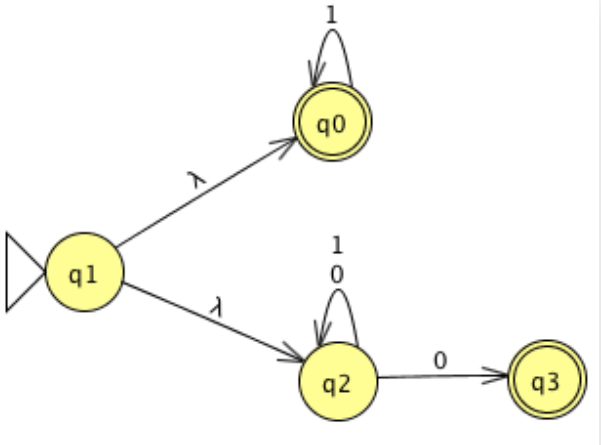
Assignment #2: Question 6, *cont'd*

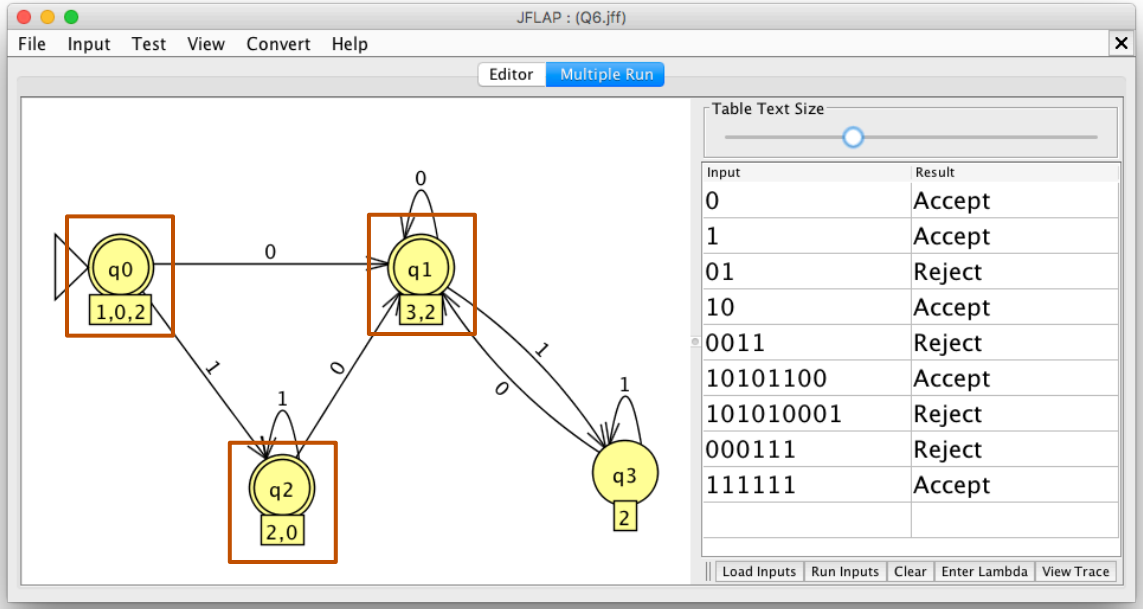




- NFA: $\delta(q_2, 0) = \{q_2, q_3\}$ so DFA $\delta(\{q_2\}, 0) = \{q_2, q_3\}$.
- NFA: $\delta(q_2, 1) = \{q_2\}$ so DFA $\delta(\{q_2\}, 1) = \{q_2\}$.

Assignment #2: Question 6, *cont'd*



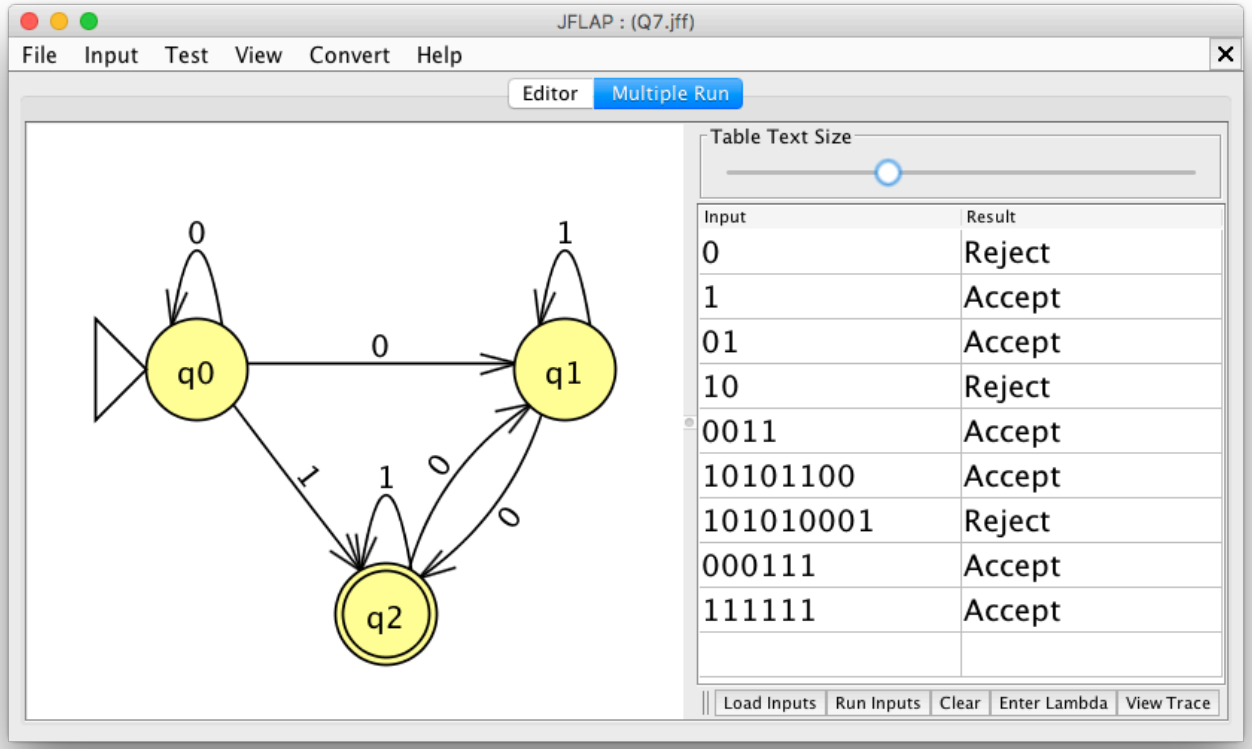


- Since NFA q_0 and q_3 are final states,
  DFA $\{q_0, q_1, q_2\}$, $\{q_0, q_2\}$, and $\{q_2, q_3\}$ must be final states.

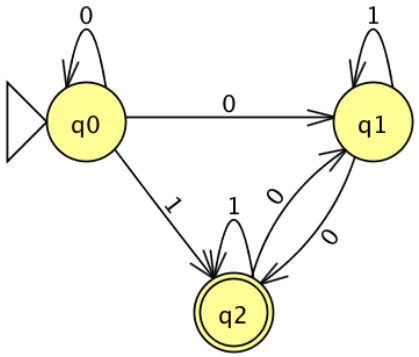
Assignment #2: Question 7

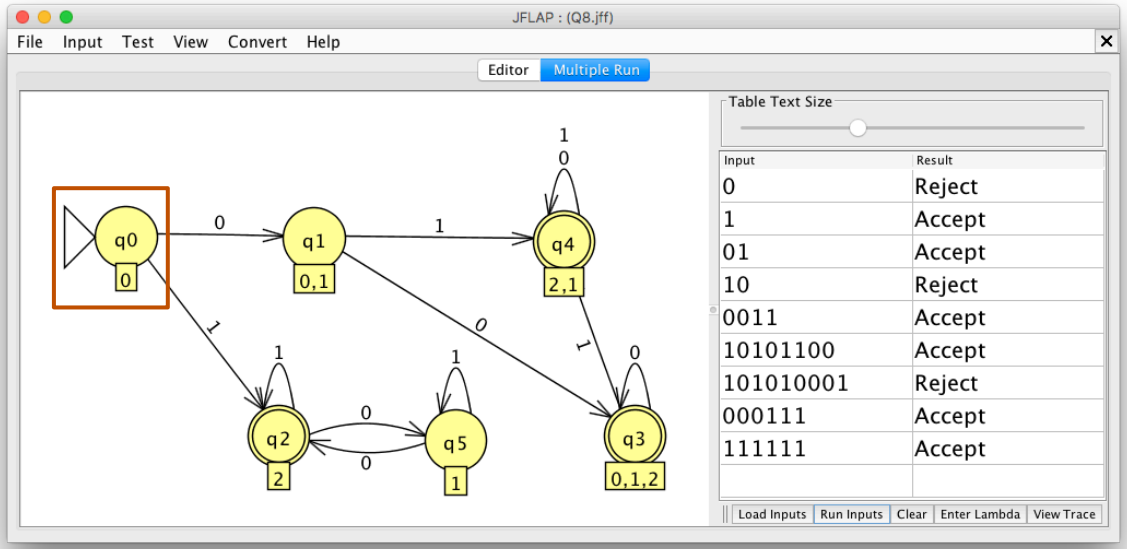
- Construct the NFA where $\Sigma = \{0, 1\}$, q_0 is the starting state, and q_2 is the final state.

	0	1
q_0	q_0, q_1	q_2
q_1	q_2	q_1
q_2	q_1	q_2



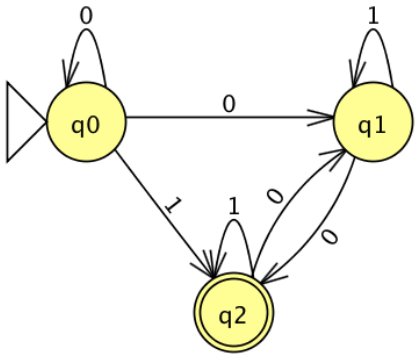
Assignment #2: Question 8

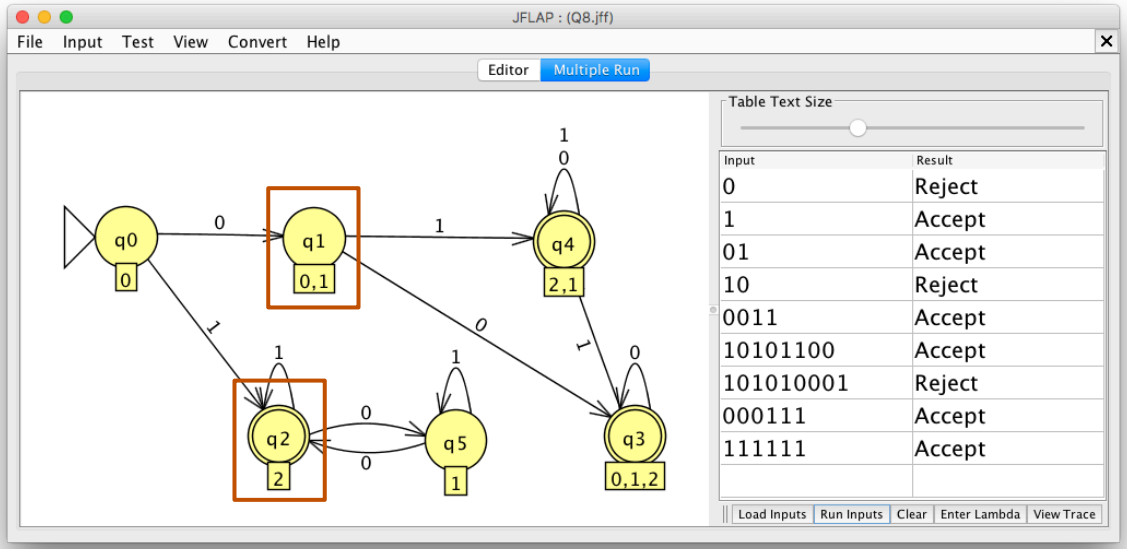




- NFA: start state q_0, so DFA start state $\{q_0\}$

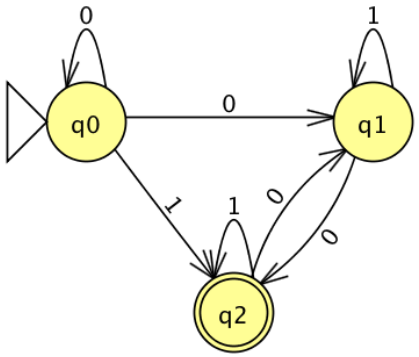
Assignment #2: Question 8, *cont'd*

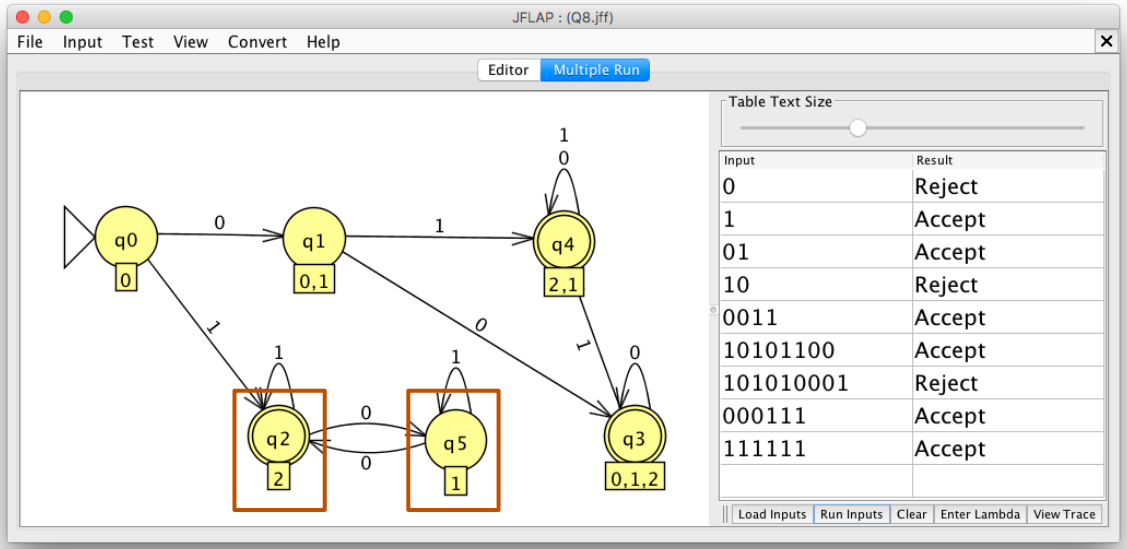




- NFA: $\delta(q_0, 0) = \{q_0, q_1\}$, so DFA $\delta(\{q_0\}, 0) = \{q_0, q_1\}$
- NFA: $\delta(q_0, 1) = \{q_2\}$, so DFA $\delta(\{q_0\}, 1) = \{q_2\}$

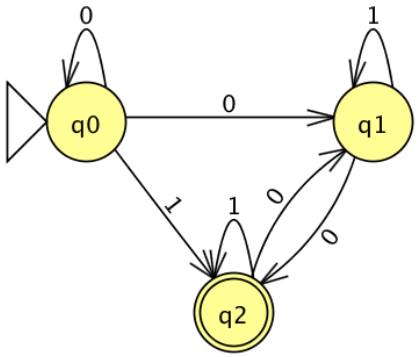
Assignment #2: Question 8, *cont'd*

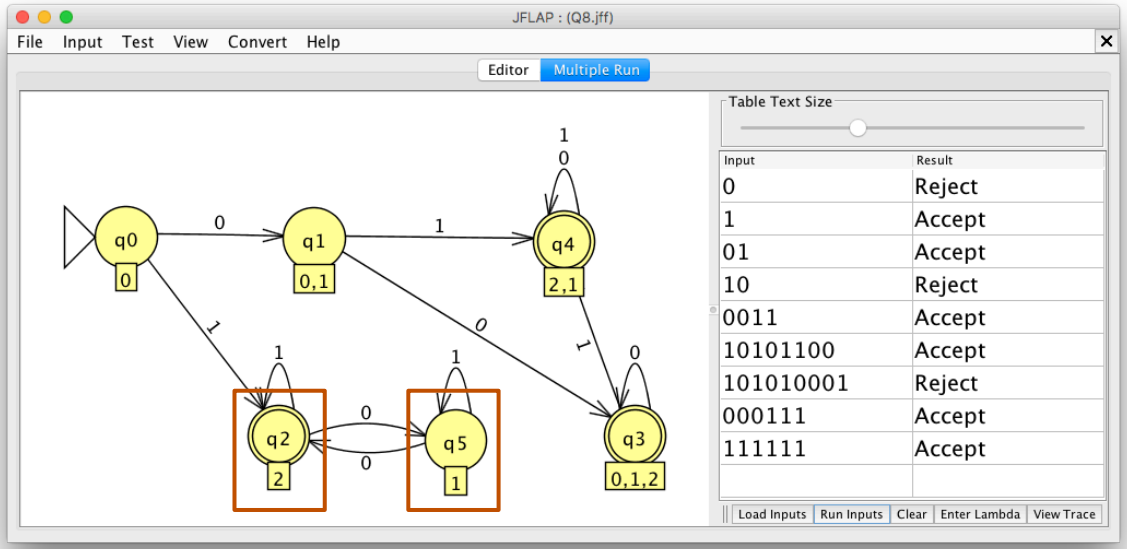




- NDA: $\delta(q_2, 0) = \{q_1\}$, so DFA $\delta(\{q_2\}, 0) = \{q_1\}$
- NDA: $\delta(q_2, 1) = \{q_2\}$, so DFA $\delta(\{q_2\}, 1) = \{q_2\}$

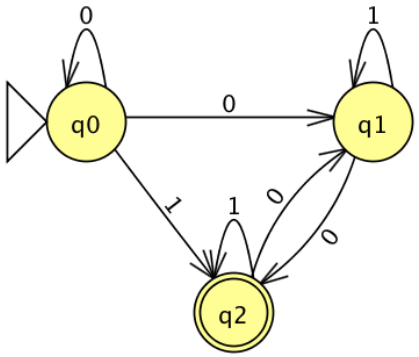
Assignment #2: Question 8, *cont'd*

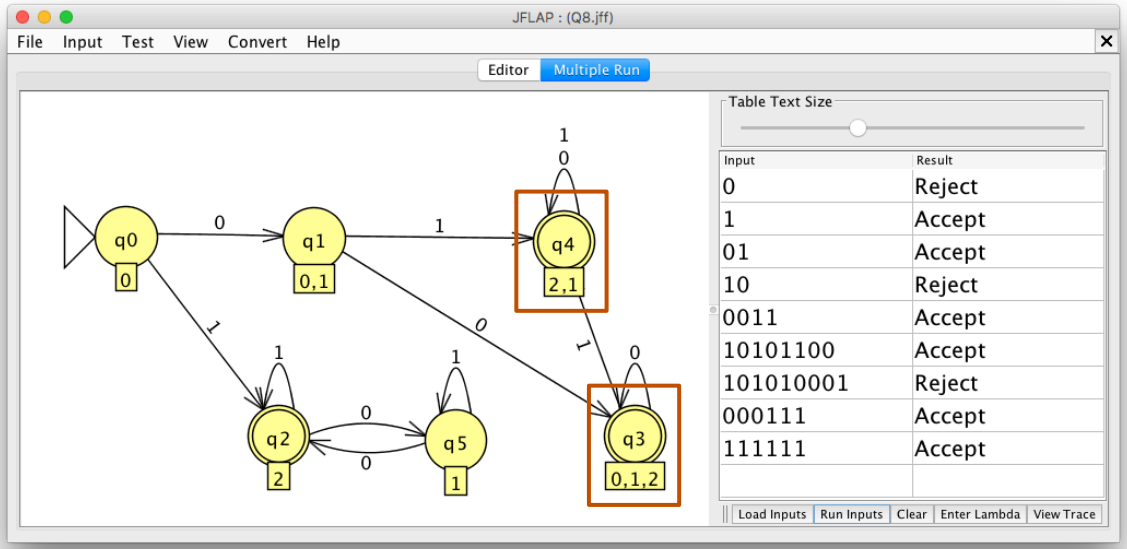




- NDA: $\delta(q_1, 0) = \{q_2\}$, so DFA $\delta(\{q_1\}, 0) = \{q_2\}$
- NDA: $\delta(q_1, 1) = \{q_1\}$, so DFA $\delta(\{q_1\}, 1) = \{q_1\}$

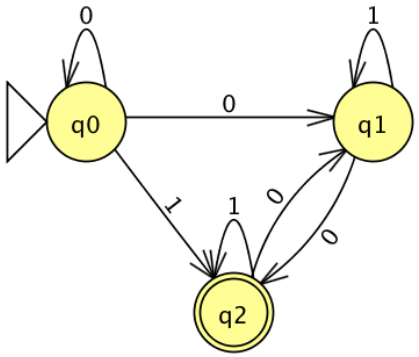
Assignment #2: Question 8, *cont'd*

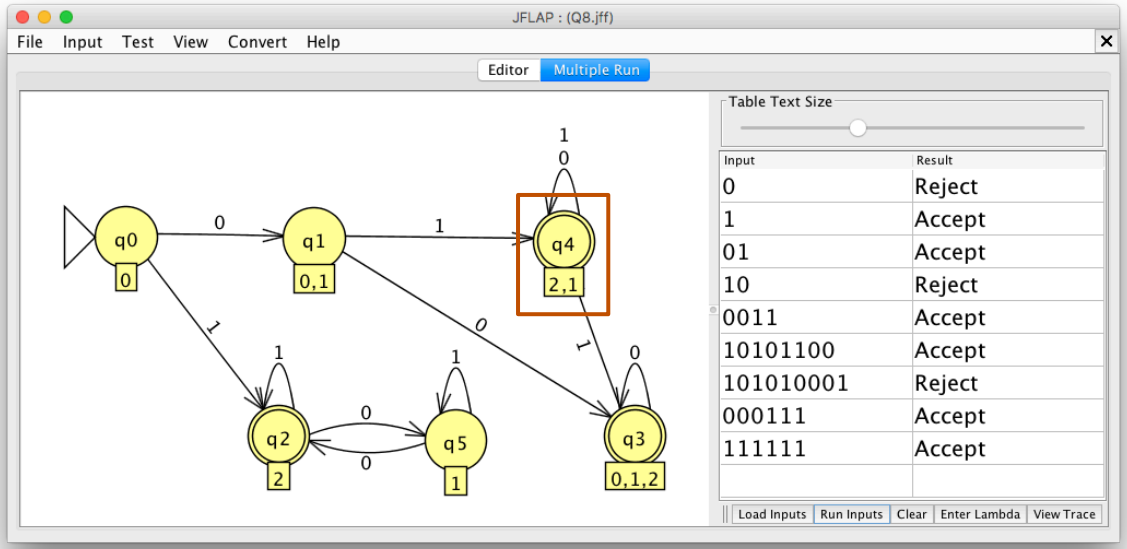




- NFA: $\delta(q_0, 0) = \{q_0, q_1\}$ and $\delta(q_1, 0) = \{q_2\}$, so DFA $\delta(\{q_0, q_1\}, 0) = \{q_0, q_1, q_2\}$
- NFA: $\delta(q_0, 1) = \{q_2\}$ and $\delta(q_1, 1) = \{q_1\}$, so DFA $\delta(\{q_0, q_1\}, 1) = \{q_1, q_2\}$

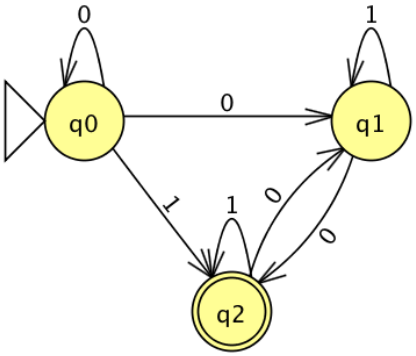
Assignment #2: Question 8, *cont'd*

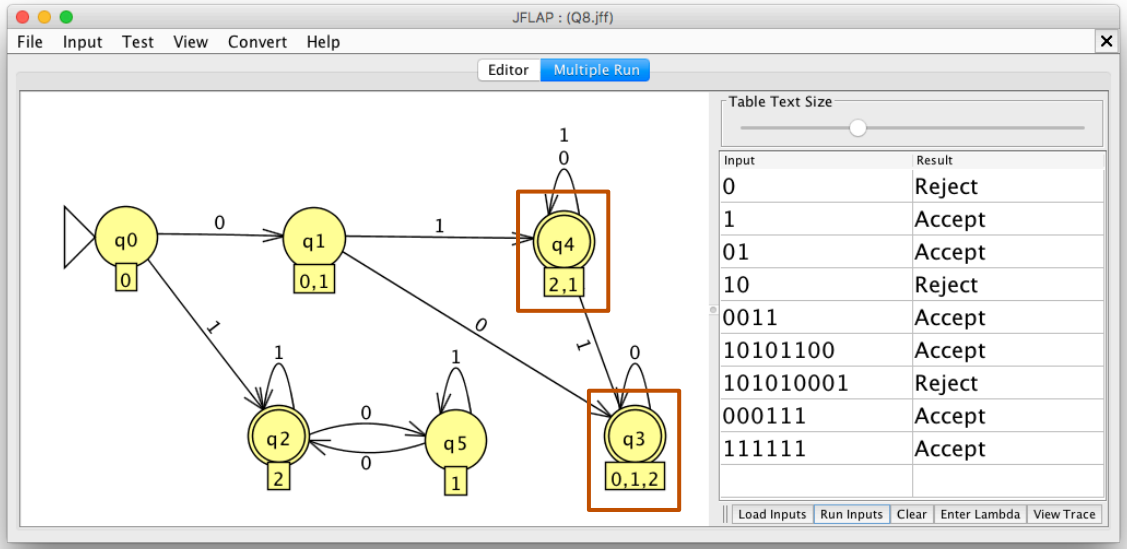




- NDA: $\delta(q_1, 0) = \{q_2\}$ and $\delta(q_2, 0) = \{q_1\}$, so DFA $\delta(\{q_1, q_2\}, 0) = \{q_1, q_2\}$
- NDA: $\delta(q_1, 1) = \{q_1\}$ and $\delta(q_2, 1) = \{q_2\}$, so DFA $\delta(\{q_1, q_2\}, 1) = \{q_1, q_2\}$

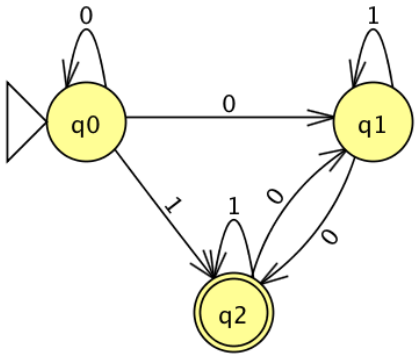
Assignment #2: Question 8, *cont'd*

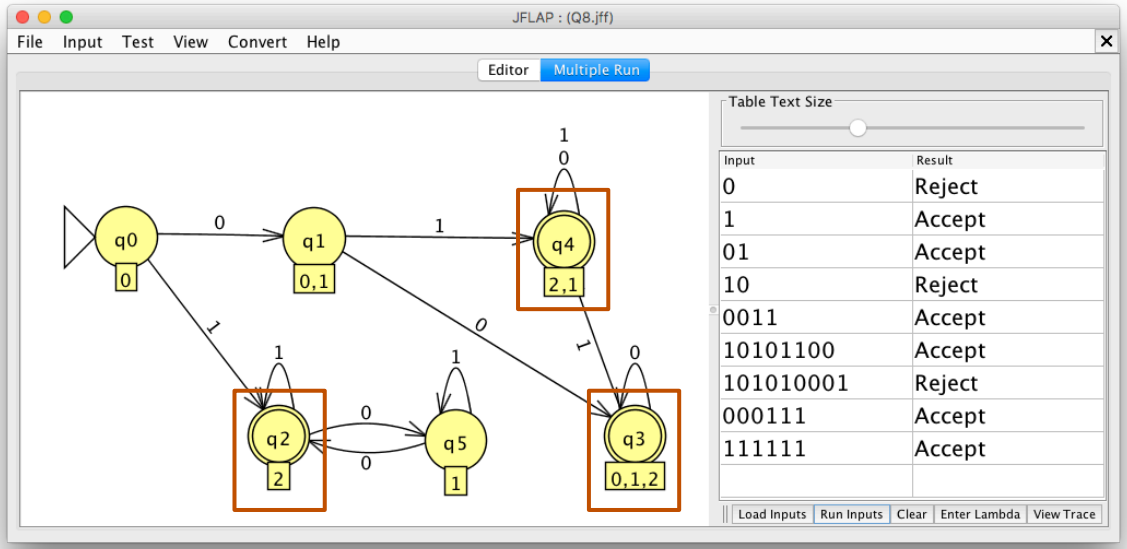




- NDA: $\delta(q_0, 0) = \{q_0, q_1\}$ and $\delta(q_1, 0) = \{q_2\}$ and $\delta(q_2, 0) = \{q_1\}$, so DFA $\delta(\{q_0, q_1, q_2\}, 0) = \{q_0, q_1, q_2\}$
- NDA: $\delta(q_0, 1) = \{q_2\}$ and $\delta(q_1, 1) = \{q_1\}$ and $\delta(q_2, 1) = \{q_2\}$, so DFA $\delta(\{q_0, q_1, q_2\}, 1) = \{q_1, q_2\}$

Assignment #2: Question 8, *cont'd*



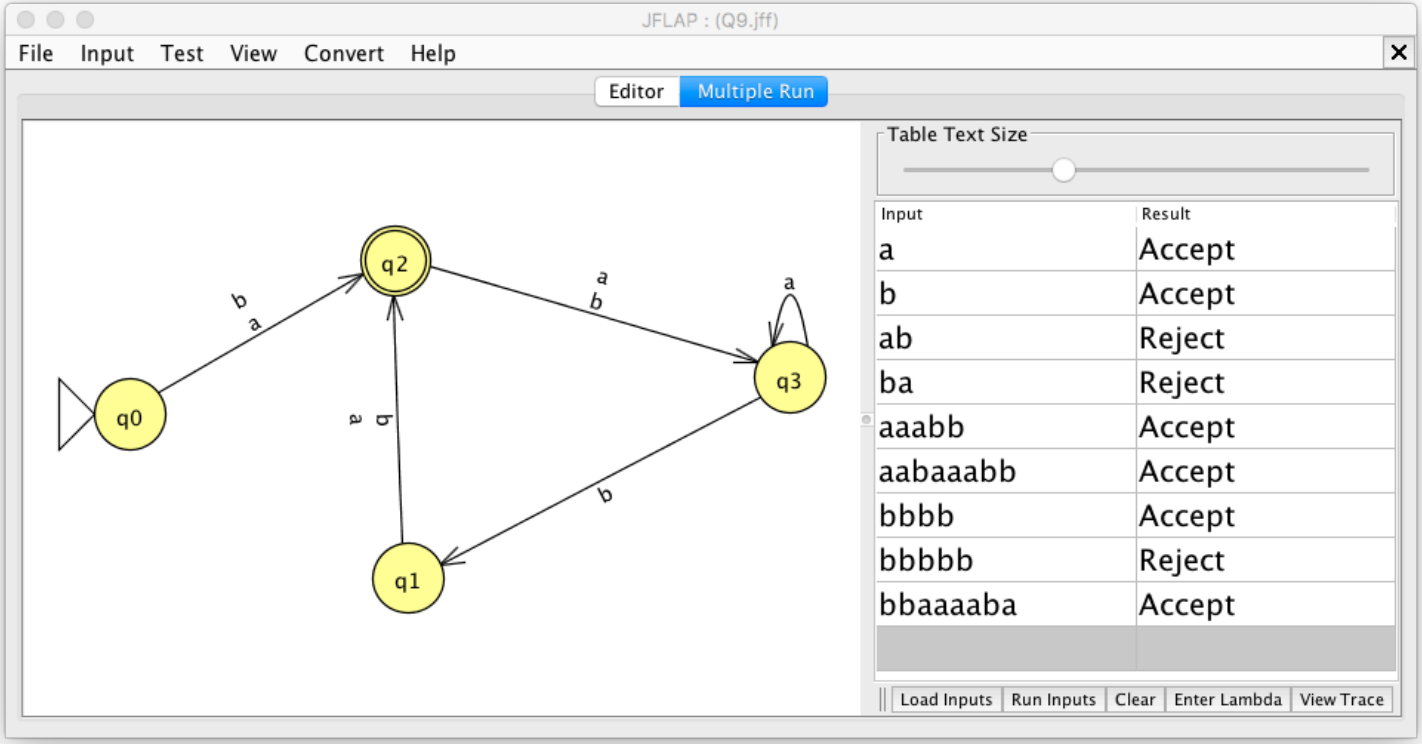


- NDA final state q_2,
  so DFA final states $\{q_2\}$, $\{q_1, q_2\}$, and $\{q_0, q_1, q_2\}$

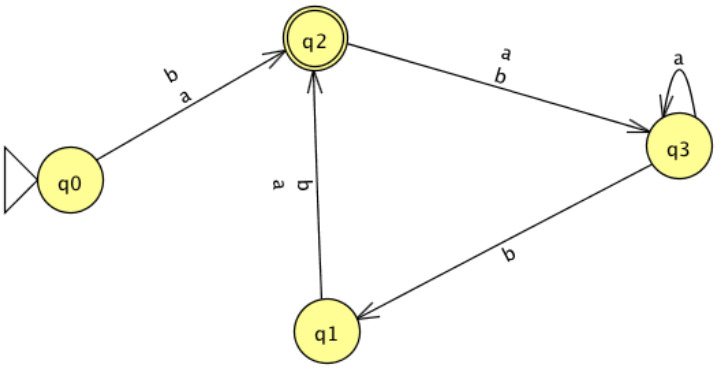
Assignment #2: Question 9

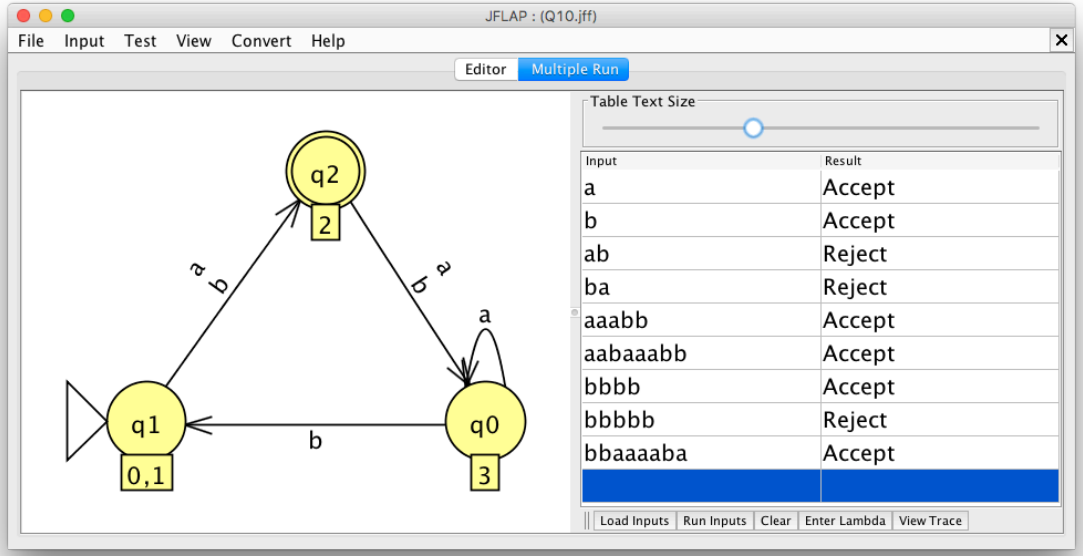
- Construct the DFA where $\Sigma = \{a, b\}$, q_0 is the starting state, and q_2 is the final state.

	a	b
q_0	q_2	q_2
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_3	q_1



Assignment #2: Question 10

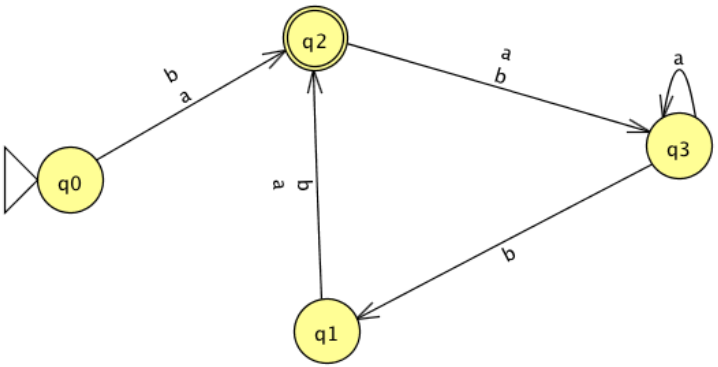


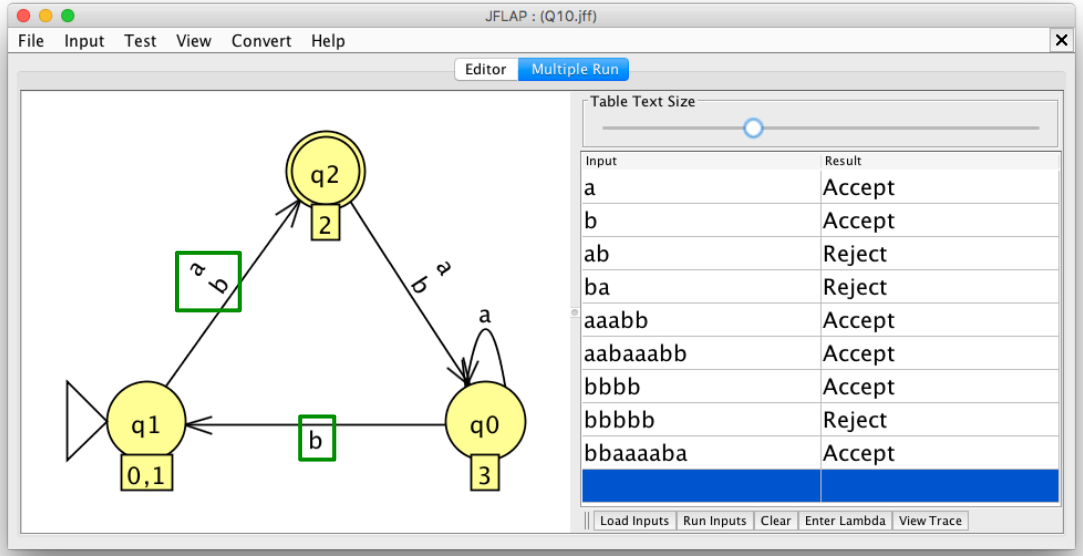


- State q_2 is final: 0 1 3 | 2
- From states q_0 and q_1, all strings lead to final state q_2:

  0 1 | 3 | 2
- No further partitioning is possible.

Assignment #2: Question 10, *cont'd*





- Original: $\delta(q_0, w) = \delta(q_1, w) = q_2$ for all w in Σ
  Minimized: $\delta^*(\{q_0, q_1\}, w) = \{q_2\}$

- Original: $\delta(q_3, b) = q_1$
  Minimized: $\delta(\{q_3\}, b) = \{q_0, q_1\}$