Homework 2

SJSU Students

CS185c

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Question 1. Homogeneous transformation matrix:

$$T = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \cos(\frac{\pi}{4}) & \sin(\frac{\pi}{4}) & -2 \\ 0 & -\sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.707 & 0.707 & -2 \\ 0 & -0.707 & 0.707 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Effects on the points (in homogeneous coordinate form):

$$T \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1.293 \\ 4.707 \\ 1 \end{bmatrix}, \qquad T \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -9.777 \\ 3.777 \\ 1 \end{bmatrix}$$

Scale by a factor of two on all axes:

$$T' = T \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 1.414 & 0.707 & -2 \\ 0 & -0.707 & 1.414 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2.

$$k' = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

$$k = \frac{k'}{\|k'\|} \approx \frac{k'}{5.92} \approx \begin{bmatrix} -0.17 \\ 0.51 \\ 0.85 \end{bmatrix}$$

$$q = \begin{bmatrix} \cos(\frac{\pi}{6}) \\ \\ \sin(\frac{\pi}{6})k \end{bmatrix} = \begin{bmatrix} \cos(\frac{\pi}{6}) \\ \\ \sin(\frac{\pi}{6}) \begin{bmatrix} -0.17 \\ 0.51 \\ 0.85 \end{bmatrix} \end{bmatrix} \approx \begin{bmatrix} 0.87 \\ -0.09 \\ 0.26 \\ 0.43 \end{bmatrix}$$

$$q^{-1} = \begin{bmatrix} 0.87 \\ 0.09 \\ -0.26 \\ -0.43 \end{bmatrix}$$

$$qvq^{-1} = \begin{bmatrix} 0.87 \\ -0.09 \\ 0.26 \\ 0.43 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0.87 \\ 0.09 \\ -0.26 \\ -0.43 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5125 \\ 0.7014 \\ -0.5298 \end{bmatrix}$$

I verified this result by creating the rotation matrix from angle and axis online, which gave me this:

$$R \cdot v = \begin{bmatrix} 0.5142857 & -0.7747822 & 0.3677265 \\ 0.6890679 & 0.6285715 & 0.3606707 \\ -0.5105836 & 0.0679007 & 0.8571429 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.514286 \\ 0.689068 \\ -0.510584 \end{bmatrix}$$

This is roughly equal to my result. The difference is explained by the early rounding to 2 digits.

Question 3. First we need to figure out the actual positions of the eyes.
This is the center of the eyes plus 3cm to the left and right of it.
Center of eyes and view direction (from the center):

$$e_c = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}, \qquad c_c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

Up vector of the eyes (which are located in XZ plane):

$$u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Find vector perpindicular to looking direction to adjust the left and right eye position:

$$r = e_c \times u = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \text{(points towards left eye from center)}$$

Calculate eye positions:

$$e_l = e_c + r * 0.03 = \begin{bmatrix} 4.97879 \\ 0 \\ 5.02121 \end{bmatrix}$$

$$e_r = e_c - r * 0.03 = \begin{bmatrix} 5.02121 \\ 0 \\ 4.97879 \end{bmatrix}$$

Calculate looking direction for both eyes:

$$c_l = -\frac{e_l}{\|e_l\|} = \begin{bmatrix} -0.7041 \\ 0 \\ -0.7101 \end{bmatrix}$$

$$c_r = -\frac{e_r}{\|e_r\|} = \begin{bmatrix} -0.7101 \\ 0 \\ -0.7041 \end{bmatrix}$$

Calculate x, y and z for each eye:

$$z_l = -c_l, \qquad z_r = -c_r$$

$$x_l = u \times z_l = \begin{bmatrix} -0.7101 \\ 0 \\ 0.7041 \end{bmatrix}, \qquad x_r = u \times z_r = \begin{bmatrix} -0.7041 \\ 0 \\ 0.7101 \end{bmatrix}$$

$$y_l = z_l \times x_l = u, \qquad y_r = z_r \times x_r = u$$

Plug value into T template:

$$T_{eye_l} = \begin{bmatrix} -0.7101 & 0 & 0.7041 & 0 \\ 0 & 1 & 0 & 0 \\ 0.7041 & 0 & 0.7101 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4.97879 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5.02121 \\ 0 & 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -0.7101 & 0 & 0.7041 & 0 \\ 0 & 1 & 0 & 0 \\ 0.7041 & 0 & 0.7101 & -7.071 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{eye_r} = \begin{bmatrix} -0.7041 & 0 & 0.7101 & 0 \\ 0 & 1 & 0 & 0 \\ 0.7101 & 0 & 0.7041 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5.02121 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4.97879 \\ 0 & 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} -0.7041 & 0 & 0.7101 & 0 \\ 0 & 1 & 0 & 0 \\ 0.7101 & 0 & 0.7041 & -7.071 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$