# Other Kinds of Dependencies 

CS157A
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## Outline

- Multivalued Dependencies and 4NF
- Join Dependencies and 5NF


## Multivalued Dependencies and 4NF

- FDs are the most common kinds of dependencies considered on the data; however, there are some other dependencies which are sometimes useful.
- For example, consider

EMP(ENAME, PNAME, DNAME)

| Smith | X | John |
| :--- | :--- | :--- |
| Smith | Y | Anna |
| Smith | X | Anna |
| Smith | Y | John |

An employee may work on several projects and have several dependents. Projects and dependents are independent of each other. so for each fixed project we must ensure each possible dependent of an employee occurs. This is called a multivalued dependency.

## Another Way to Think About FDs AND MVDs

- Suppose we had an FD A-->BC on a table T with columns ABC.
- One was to think of this dependency is as two maps $\mathrm{F}_{1}: \operatorname{dom}(\mathrm{A})$--> $\operatorname{dom}(\mathrm{B})$ and $\mathrm{F}_{2}: \operatorname{dom}(\mathrm{A})-->\operatorname{dom}(\mathrm{C})$, such that every row in $T$ can be written as $\left(a, \mathrm{~F}_{1}(\mathrm{a}), \mathrm{F}_{2}(\mathrm{a})\right)$ for some $\mathrm{a} \in \operatorname{dom}(\mathrm{A})$.
- One way to generalize this would be to allow our functions to map to subsets of dom(B) and dom(C) rather than to just one value. Hence, our functions would become multivalued.
- So now $\mathrm{F}_{1}(\mathrm{a})$ might output $\left\{\mathrm{b}_{1}, \ldots, \mathrm{~b}_{\mathrm{m}}\right\}$ and $\mathrm{F}_{2}(\mathrm{a})$ might output $\left\{\mathrm{c}_{1}, \ldots\right.$, $\left.c_{n}\right\}$.
- To store all the information with associated with $a$ in the original table, which since we are in 1NF cannot store multivalued data, we need for each possible $i$ and $j$, a row ( $\mathrm{a}, \mathrm{b}_{\mathrm{i}}, \mathrm{c}_{\mathrm{j}}$ ).

Formal Definition of Multivalued Dependency

Definition: A multivalued dependency (MVD) X-->> Y holds on R means that if two distinct tuples t 1 and t 2 exist in r such that $\mathrm{t} 1[\mathrm{X}]=\mathrm{t} 2[\mathrm{X}]$ then two tuples t 3 and t 4 should also exist in r with the following properties: (let Z denote $\mathrm{R}-(\mathrm{X} \cup \mathrm{Y})$ )
$-\mathrm{t} 3[\mathrm{X}]=\mathrm{t} 4[\mathrm{X}]=\mathrm{t} 1[\mathrm{X}]=\mathrm{t} 2[\mathrm{X}]$
$-\mathrm{t} 3[\mathrm{Y}]=\mathrm{t} 1[\mathrm{Y}]$ and $\mathrm{t} 4[\mathrm{Y}]=\mathrm{t} 2[\mathrm{Y}]$
$-\mathrm{t} 3[\mathrm{Z}]=\mathrm{t} 2[\mathrm{Z}]$ and $\mathrm{t} 4[\mathrm{Z}]=\mathrm{t} 1[\mathrm{Z}]$

- Notice if $X--\gg Y$ then also $X--\gg Z$
- Notice any FD is an MVD.
- If Y is a subset of X then the MVD is trivial.
- MVDs also have rules of inference like Armstrong's axioms (see book).


## 4NF

Definition: A relation schema R is in 4NF with respect to a set of FDs and/or MVDs F if, for every nontrivial multivalued dependency $\mathrm{X}--\gg \mathrm{Y}$ in $\mathrm{F}^{+}$, X is superkey for R .

## Binary Test for Multivalued Lossless Joins

- The binary test for LJP we had for functional dependencies can be modified to multivalued dependencies:
- Lossless Join Test:

The schemas $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ form a lossless join decomposition of R with respect to a set F of FDs and MVDs iff:

$$
\left(R_{1} \cap R_{2}\right) \rightarrow\left(R_{1}-R_{2}\right)
$$

or

$$
\left(R_{1} \cap R_{2}\right) \rightarrow\left(R_{2}-R_{1}\right)
$$

## Lossless Join Decomposition into 4NF

Input: A universal relation R and a set of MVDs F on the attributes of $R$
Set D: $=\{\mathrm{R}\}$;
While there is a relation Q in D that is not in 4 NF
\{
1.choose a relation Q in D not in 4NF
2.find a MVD X-->>Y in Q that violates 4NF
3.replace Q in D by the two relations $(\mathrm{Q}-\mathrm{Y})$ and $(\mathrm{X} \cup \mathrm{Y})$
\}
Since 4NF is essentially BCNF rewritten for MVDs, this
algorithm works for the same reason our BCNF algorithm worked.

## Join Dependencies

- Our lossless join conditions for FDs and MVDs tell us when we can replace a schema R with two schema R1 and R2 and still get a lossless join.
- Sometimes there may be no way to split R into just two relation R1 and R2 and still get the LJP to hold.
- Nevertheless, one might be able to split R into $\mathrm{R}_{1}, \ldots \mathrm{R}_{\mathrm{n}}$ where $\mathrm{n}>2$ and get LJP to hold.
- We say a join dependency, $\mathrm{JD}\left(\mathrm{R}_{1}, \ldots \mathrm{R}_{\mathrm{n}}\right)$, holds among $\mathrm{R}_{1}, \ldots \mathrm{R}_{\mathrm{n}}$, a decomposition of R , if for every legal r of R :

$$
\pi_{R_{1}}(r) * \pi_{R_{2}}(r) * \cdots * \pi_{R_{n}}(r)=r
$$

- It turns our any MVD can be written a s JD.
- A JD is called trivial if one of the $\mathrm{R}_{\mathrm{i}}$ equals R .


## 5NF

Definition: A relation schema is in $\mathbf{5 N F}$ (or project-join normal form) with respect to a set F of FDs, MVDs, and JD if, for every nontrivial join dependency $\operatorname{JD}\left(\mathrm{R}_{1}, \ldots \mathrm{R}_{\mathrm{n}}\right)$ in $\mathrm{F}^{+}$, every $\mathrm{R}_{\mathrm{i}}$ is a superkey of R .

The book gives an example of a table SUPPLY(SNAME, PARTNAME, PROJNAME).
for a supplier who supplies parts to a project. The data in the book could be decomposed into three tables R1(SNAME, PARTNAME), R2(SNAME, PROJNAME), and R3(PARTNAME, PROJNAME) with the LJP. However, decomposition into any two tables would not preserve this property.

