1st Order Logic

Giving meaning to 1st order formulas:

Need to have a set M called the universe that our variables range over.
For each constant c in our language, we need to give a value C^m in M.
For each function f in our language need to give an actual function determined on f^m

For each predicate symbol P
P^m:M -> {T, ⊥}

All of the above information together is called a structure/model. Usually denoted by ***M***.

Can almost now talk about the meaning of a formula F in a given structure ***M***.

Last thing, in F(x1, …, xn):
(Exists x)G(x, y) x is a bound variable, y is unbound

A variable/object assignment v is a map from unbound variables to elements of M.

Write ***M*** |= F[v] (***M*** entails F with assignment v)

What this means…
If F is an atomic formula calculate the value in M of each of the terms in F.
Plug these values into the predicate P^m and see if outputs T or F (⊥)

If F : = NOT G then F is true in M, v iff not M |= G[v]

If F: = G OR H Then M |= F[v] iff M |= G[v] or M |= H[v]

If F := G AND H then M |= F[v] iff M |= G[v] and M |= H[v]

If F := (Exists x) G then M |= F[v] iff there is some way to map x -> x^m in M such that v is extended by this additional mapping give M |= G [x-> x^m, v]

If F := (Forall x) G then M |= F[v] iff for all ways to map x -> x^m in M
M |= G [x->x^m , v]

Write M |= F if for every variable assignment, v, M |= F[v]

In AI had a notion of a knowledge base entailing a formula
Write KB |= F to mean for all structures M, such that M |= KB
we also have M |= F

Proofs in 1st Order Logic

Very much like proofs in propositional logic except now we have some additional axioms.

Example additional axioms

(Forall x) (NOT P) => NOT((Exists x) P)
(for every pig, it can't fly = not there is a pig that can fly)

NOT(Forall x P) =>(Exists x) NOT P
(Not for every horse it is blue = there is a horse that is not blue)

A(t) => (exists x) A(x)
A(x)=> (forall y) A(y)

(A AND B) => A

etc.

Example 1st Order Knowledge bases

a) Any relational database.
b) Set theory
Has one constant 0 (empty)
and predicate symbols Є (element of), =
No function symbols

Knowledge base for set theory consists of formulas like
NOT(0 Є 0)
Exists x NOT(x = 0)