## CS154 Midterm Spring 2019 Version 1

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## Instructions:

1. This midterm is due $11: 59 \mathrm{pm}$, March 12 .
2. To complete the midterm, print it out, fill in your answers on the midterm, and scan it back into a file midterm.pdf where the total size is less than 10 MB .
3. If you don't have a printer, copy and paste each problem into a word processor document. Then after each problem write your solution. Make a less than 10 MB midterm.pdf file of the result and submit that.
4. Use the same submit mechanism as for the homeworks to submit your completed midterm.
5. Each problem on this midterm is worth the same amount (4pts).
6. If you have a question on the interpretation of a problem on the midterm, you can email me at chris@ pollett.org.
7. Due to the coronavirus this is an open book, open internet midterm.
a. What that means is that you can consult any static (on the order of static for weeks) source of information related to the midterm material.
b. You cannot directly or indirectly ask another person how to do any problem off the midterm.
c. To receive credit on problems that make use of your personal information, you need to have correctly filled in that personal information.
d. When you submit your completed midterm, you are asserting all of the work in the midterm is your own.
8. Use the algorithms from class (a) to step-by-step convert the regular expression $(00 \cup 11)^{\star}$ to an NFA (2pts). (b) convert this NFA to a DFA (you only need to show portion reachable from start state) (2pts).
9. Let $m=$ the fourth leftmost digit of your StudentID + 5. I.e., if your ID was 123456789 , then $m=4+5=9$. Do the following: (a) Show the union of $m$ countable sets is countable (1pt), (b) compute the m'th successor set of $\left\{\},\{\{ \}\}\}(1 \mathrm{pt})\right.$, (c) express the m-tuple $\left(a_{1}, a_{2}, \ldots, a_{m}\right)$ as a set using iterated pairing (1pt), (d) give one (not all) possible partition of the set $\left\{a_{1}, a_{2}, \ldots, a_{m}\right\}$ (1pt).
10. Let $n$ be your Student ID. Prove there is a language $L$ (over the alphabet of your choice) with exactly $n$ many distinct equivalence classes $[w]_{L}$ with respect to the string distinguishability. (2pts). Give the minimal DFA for your language. (2pts).
11. Let $\mathrm{k}=$ the rightmost digit of your StudentID +16 in binary. I.e., if your ID was 123456789 . Then $k=(9+16)_{2}=(25)_{2}=\left(2^{4}+2^{3}+2^{0}\right)_{2}=11001$. Consider the homomorphism given by $h(0)=k, h(1)=1$. (a) Give a regular expression $R$ for the language over $\{0,1\}$ that consists of strings with two or more occurrences of 0 . (b) Using the closure under homomorphism construction from class, give the regular expression for the language $h(L(R))(1 \mathrm{pt})$. (c) prove using the pumping lemma that the language $\left\{k^{n} 1^{n} \mid n \geq 0\right\}$ is not regular. (2pts).
12. Let $\mathrm{k}=$ the rightmost digit of your StudentID +16 in binary. Given two languages $L_{1}$ and $L_{2}$ over alphabet $\{0,1\}$, define the less- $k$, difference of $L_{2}$ from $L_{1}$ to be the set $\left(L_{1}-\left(\{k\} \cup L_{2}\right)\right)$. Suppose we have DFAs $M_{1}$ and $M_{2}$ that recognize $L_{1}$ and $L_{2}$. Use these machines and product constructions to show that the less- $k$, difference of $L_{2}$ from $L_{1}$ is regular.
