CS154 Final Spring 2020

Name:
StudID:

Instructions:

1. This final is due midnight, May 14.
2. To complete the final, print it out, fill in your answers on the final, and scan it back into a file Final.pdf where the total size is less than 10MB.
3. If you don't have a printer, copy and paste each problem into a word processor document. Then after each problem write your solution. Make a less than 10MB Final.pdf file of the result and submit that.
4. Use the same submit mechanism as for the homeworks to submit your completed final.
5. Each problem on this final is worth the same amount (3pts).
6. If you have a question on the interpretation of a problem on the final, you can email me at chris@pollett.org.
7. Due to the coronavirus this is an open book, open internet final.
    a. What that means is that you can consult any static (on the order of static for weeks) source of information related to the final material.
    b. You cannot directly or indirectly ask another person how to do any problem off the final.
    c. To receive credit on problems that make use of your personal information, you need to have correctly filled in that personal information.
    d. When you submit your completed final, you are asserting all of the work in the final is your own.

Problem	Grade	Problem	Grade
1		6	
2		7	
3		8	
4		9	
5		10	

1. State precisely what the Halting problem is (0.5pt). Let H be the language related to the halting problem consisting of strings $\langle M, w\rangle$ such that when machine M is started on binary string w, M halts and the final tape contents is how old you are on the exam due date written in binary. Show H is undecidable using an argument via a Turing reduction (1pt). Show H is undecidable using an argument using Rice's Theorem (0.5pt). Come up with a scaled down version of H which is NP-complete (show with proof) (1pt).

2. Let L_w be the language over { '(', ')', a,b,...,z, A, B,..., Z} consisting of a correctly parenthesized string of only '('s and ')'s followed by the string w. So for example, the following strings are in $L_{Pollett}$: Pollett, ()Pollett, ()()Pollett, ((()))Pollett; whereas, ()()(, (Pollett), ()(Pollett, are not. Let w be the string corresponding to your last name. Give a CFG for L_w that has an epsilon rule, a unit rule and at least one rule of with more than two right hand symbols (1pt). Show step-by-step how to convert it to Chomsky Normal Form (1pt). Show step-by-step how the CYK algorithm would work for your grammar on the string ()w (1pt).

3. Let m = the year you were born. Show how the SEQUITUR algorithm would compress the string m^5 (2pts). For Professor Pollett, $m^5 = 19701970197019701970$. Give the formal description of a TM recognizing only the string m^5 (1pt).

4. Give a CFG for a language involving your birthday that is ambiguous (with proof that it is ambiguous) (1pt). Prove the language $L = \left\{ www \mid w \in \{0, 1\}^\star \right\}$ is not CFL (2pts).

5. Let m = the seventh leftmost digit of your StudentID + 1 written in binary. I.e., if your ID was 123456789, then $(m)_2 = (7+1)_2 = (8)_2 = 1000$. Let $\overline{(m)_2}$ be the binary string where each 0 in $(m)_2$ is changed to a 1 and each 1 in $(m)_2$ is changed to a 0. So for the example given before, $\overline{(m)_2} = 0111$. Define a function h such that $h(0) = (m)_2$ and $h(1) = \overline{(m)_2}$, and for a string $w = w_1...w_n$ over $\{0, 1\}$, $h(w) = h(w_1)...h(w_n)$. Give a regular expression for the language L consisting of binary strings with at least three 0's (1pt). Using the closure of regular languages under homomorphism construction, give a regular expression for $\{h(w) \mid w \in L\}$ (2pts).

6. Let m = the fourth leftmost digit of your StudentID + 4 written in binary. I.e., if your ID was 123456789, then $(m)_2 = (4+4)_2 = (8)_2 = 1000$. Consider the language $L = \left\{w \mid w \in \{0,1\}^\star \text{ has an even number of substrings of the form } (m)_2\right\}$. Give an $O(n)$ algorithm written in Java (that compiles), for determining if a string is in the language L. (0.5pts, basic algorithm, 1pt algorithm implemented based on some automata that recognizes L). Prove it is $O(n)$ up to a set of justifiable assumptions that you state. We need some assumptions because of things like int's in Java being 32-bit, etc. (1pt proof, 0.5pts stated assumptions).

7. Let m = the fifth leftmost digit of your StudentID + 2. I.e., if your ID was 123456789, then $m = 5 + 2 = 7$. Prove carefully that the language $\left\{0^{m^n} \mid n \geq 0\right\}$ is not regular (3pts). *Hint: the idea is in the notes, but does not use the pumping lemma*.

8. Explain how the DFA state minimization algorithm from class works (2pts). Give a concrete example of using it. (1pt).

9. Define the term computation history (1pt). Prove E_{LBA} is undecidable using a computation history argument (2pts).

10. Draw a PDA capable of doing palindrome checking for string over the alphabet $\{a, b, c\}$ (3pts).