The test will be open book, open notes, 2 hours and 15 minutes time limit. Please write your answers on the exam sheet. Seventeen questions, six points per problem, total 92 points possible.

If a problem involves an alphabet, unless otherwise specified that alphabet consists of letters mentioned in the problem only, for example, just \{a,b\} if no other letter is mentioned, or just \{a,b,c\} if those letters are mentioned.

Throughout the exam the acronyms “PDA” and “npda” are synonymous. The start symbol of all our grammars is $S$ and the top-of-stack symbol is always $Z$. The start state of any kind of machine (in this exam) is always $q_0$.

1. Exhibit a DFA accepting exactly this language: The set of strings over the alphabet \{a,b,c\} without any double letters. For example, $abcabc$ is in this language, but $abba$ is not, and $abcba$ is not.
2. Give a regular expression \( p \) such that \( L(p) \) is the language accepted by the following NFA:

![NFA Diagram]

*Answer:*

3. Find a finite automaton (NFA is OK) accepting exactly the strings matching this regular expression: \( a^*b + b^*a \)
4. Convert the following NFA to a DFA
5. Minimize the following DFA:
6. Which of these languages are regular? *Circle the letters of those that are regular.*

(a) The set of strings in the two letters ‘(’ and ‘)’ with incorrectly matched parentheses.

(b) The set of strings that look like they could be social security numbers.

(c) The set of all California driver’s license numbers issued prior to Dec. 31, 2007.

(d) The set of all codes of non-deterministic Turing machines (coded by the coding in the textbook).

(e) The set of strings with more $a$’s than $b$’s

(f) The set of strings whose length is divisible by 7

7. Write a grammar that generates the set of palindromes over \{a,b\} whose length is divisible by 6. For example, \textit{abaaba} is in this language, and so is \textit{abaabbbbaaba}, but \textit{ababa} is not and \textit{abab} is not. The empty string is in this language since 0 is divisible by 6.

8. Exhibit a pdas accepting the following language: \{ $a^{2n} b^{2n} : n > 0 \} You can give your answer as a transition diagram or as a transition function $\delta$. 
9. Exhibit a PDA that accepts exactly the strings generated by the following grammar.

\[ S \rightarrow aSc \mid cBa \mid ac \]

\[ B \rightarrow bB \mid b \]

10. Construct a Turing machine accepting the set of strings on the alphabet \{a,b\} of odd length.
11. True or false questions about the halting problem.

(a) There is a Turing machine that accepts exactly the set of codes of Turing machines.

(b) There is a Turing machine that accepts exactly the set of codes of Turing machines that do not accept their own codes.

(c) There is a Turing machine that accepts exactly the set of codes of Turing machines that accept their own codes.

(d) There is a Turing machine $M$ such that, for every Turing machine code $e$, $M$ halts and outputs 1 if the machine with code $e$ accepts its own code, and 0 if not.

(e) There is a Java program that takes a Turing machine code $e$ as input (as a String) and simulates that Turing machine (at input “elephant”) writing its output using System.out.println if it halts, and never halting unless/until the machine with code $e$ halts.

(f) There is a Java program that takes a Turing machine code $e$ as input (as a String), and tells us, in a finite amount of time, whether or not the machine with code $e$ will halt (at input “elephant”).

12. Prove that the set of Turing machine codes $e$ such that machine $e$ halts at input “giraffe” is not Turing computable.
13. For each of the following sets, choose one of the following labels: recursive (R), recursively enumerable (RE), or neither (N). Labeling a set RE means that you think there is no obvious reason why it should be recursive (it might still actually be recursive due to some complicated mathematics yet to be discovered); similarly for N.

(a) The set of strings over the alphabet \{a,b,c\} that have equal numbers of all three symbols.

(b) The set of codes of Turing machines.

(c) The set of integers \(n\) such that the sequence you get by starting with \(n\), and repeatedly either multiplying by 3 and adding 1 (for odd numbers) or dividing by 2 (for even numbers) eventually reaches 1.

(d) The set of Turing machines whose output (at blank input) is 80.

(e) The set of Turing machine codes \(e\) such that for some input, the machine with code \(e\) does not halt.

14. Consider a Turing machine defined as follows: if in state \(q_1\), reading \(a\), stay in \(q_1\), write \(b\), and move right. If in state \(q_1\), reading \(b\), stay in \(q_1\), write \(a\), and move right. (So this machine runs through its input changing \(a\) to \(b\) and \(b\) to \(a\).) Exhibit the code for this machine according to the coding scheme given in your textbook:

\(Answer:\)
15. True or false (for sets of strings over some alphabet).

(a) Every r.e. set is recursive

(b) Every r.e. set is generated by a context-free grammar

(c) The complement of a recursive set is recursive

(d) The complement of an r.e. set is r.e.

(e) There are r.e. sets that are not recursive.

(f) There are recursive sets that are not r.e.

16. What is meant by saying that Turing machine M “solves the halting problem”?

This question is answered in three parts, by describing the valid inputs to M, when the machine would have to halt (produce an output), and what the outputs would have to be.

(a) What would constitute valid input to such a machine? (In other words, invalid input is input on which it is irrelevant what the machine does).

(b) On which valid inputs would the machine be required to halt?

(c) If the input has the form you specified in (a), and satisfies the condition you specified in (b), what would the output be required to be?

17. Let $\Sigma$ be an alphabet, and let $f$ be a function from $\Sigma^*$ to $\Sigma^*$, i.e. the inputs to $f$ and the values of $f$ are both strings over $\Sigma$. Explain what it means for Turing machine M to compute $f$. 