Pumping Lemma Exercises
CS154 Assignment 7

In each of these problems, you are given a language (set of strings). Your task is to use the pumping lemma to prove that the language in that problem is not regular. Follow the template given in class for such proofs. Scoring, two points per problem, total 10 as usual.

If you attach extra sheets of paper, please staple everything together securely (no paper clips or dogears).

1. Let the alphabet consist of the two characters ‘(’ and ‘)’ (left and right parentheses). Let the language be the set of strings with correctly matched parentheses. The exact definition of “correctly matched parenthesis” is the same here as in programming languages. For example (()()) and (((())())()) belong to this language but ((()) does not.

Remark. This example should surely convince you that we need more tools than just finite automata and regular expressions to write compilers.
2. Let the alphabet be \(\{a, b\}\) and the language be the set of strings with twice as many \(a\)'s as \(b\)'s.

3. Let the alphabet be \(\{a, b\}\) and the language be the set of strings with more \(a\)'s than \(b\)'s.
4. Let the alphabet consist of just one letter $a$. Let the language be the set of strings whose length is a prime number.
5. Let the alphabet be the set of symbols with ASCII codes, and the language the set of compilable Java programs (regarding each program as a single string, the contents of a Java file.) *Hint:* it is legal to declare an integer-valued function of an integer argument, and then write things like \( f(f(f(f(f(n)))))) \). In that way you can make this problem somewhat like problem 1. No doubt there are also other solutions.