Then one can check that $f$ induces an edge-graceful labeling.

Example 10. Edge-graceful labelings for $Sp(3:2)$, $Sp(5:2)$, $Sp(4:2)$, and $Sp(6:2)$ (Figure 21). There are other odd spiders that are proved to be edge-graceful, see Lee, Small, Yao and Huang [6].

5. Odd Trees Constructed from the Amalgamation of Complete $2m$-ary Trees

Let $T_{2m,n}$ be a complete $2m$-ary tree of $n$ levels. Lo [7] showed that $T_{2m,n}$ is edge-graceful. For any $m \geq 1$, let $T(2m)$ be the class of all complete $2m$-ary trees. We denote $ET = \cup \{T(2m) : m \geq 1\}$. Given a finite collection of trees, say $T_1, \ldots, T_n$ in $ET$, we can construct a new tree by amalgamating $T_1, \ldots, T_n$ with their roots, i.e., union $T_1, \ldots, T_n$ identify the roots. If we take $k$ isomorphic copies of $T$ and identify all the copies with their roots, then the resulting graph will be denoted hereafter by $T_{2m,n}^{(k)}$ and it is an odd tree.

Example 11. $T_{2,3}^{(4)}$ (Figure 22).

Theorem 8. For any finite collection $\{T_i : i \in I\}$ of rooted trees in $ET$, the odd tree which is constructed by amalgamation is edge-graceful.

Corollary 9. The odd tree $T_{2m,n}^{(k)}$ is edge-graceful for any $k \geq 1$, $m \geq 1$.

The above results follow from the more general result which we will prove in the next section.