The graph $G = C_n \times P_m$ has diameter $\lfloor n/2 \rfloor + (m-1)$. For $n \geq 4$, we see that $D(G) > 3$.

To see that $G$ is diameter $d$-invariant. Consider the edge $e = (v_{i,1}, v_{i+1,1})$ where $1 \leq i \leq n$ or $(v_{i,1}, v_{i,2})$ for $1 \leq i \leq n$. Without loss of generality, assume $e = (v_{i,1}, v_{2,1})$. We see that $d_G e (v_{i,1}, v_{2,1}) = 3$ and $d_G (u, v) = d_G e (u, v)$ for all other edge $(u, v) \neq (v_{i,1}, v_{2,1})$. Therefore $D(G\setminus e) = D(G)$. Similarly, for $e = (v_{i,1}, v_{i,2})$ we also have $D(G\setminus e) = D(G)$. Thus $G$ is d.e.i.

5. **Edge Expansion Construction.**

Let $2$-Gph be the class of all d.e.i. graphs $H$ with two terminals $(u, v)$. If $<H; (u, v)>$ with $d(u, v) = D(H)$, then we say it is *diametric* (Note this terminology come from [12]).

Let $G$ be a directed graph whose underlying graph $G^*$ is connected and for any mapping $f : E(G) \rightarrow 2$-Gph with $f(e) = <H; (u, v)>$ we can construct a new undirected graph by replaced all the directed edges $e = (a, b)$ of $G$ by $H$ with identification of vertex $a$ with $u$ and vertex $b$ with $v$. The resulting graph is called an edge expansion of $G$ by $f$ and will denoted by $G[f]$.

In particular, if $<H; (u, v)>$ is diametric and $f(e) = <H; (u, v)>$, for all $e$ in $G$, then we shall use $G[H; u, v]$ to denote the edge expansion of $G$ by $f$.

Let us describe the above construction by the following example.

**Example 3.** Let $G$ be a directed cycle of length 3 and the mapping $f : (e_1, e_2, e_3) \rightarrow (H_1, H_2, H_3)$ with $f(e_1) = H_1$ for