a d.e.i. graph with diameter \( k \geq 2 \).

**Proof.** Take a graph \( G \), with diameter \( k \). Consider \( G[H] \) which is diameter e-invariant by Corollary 5.

This result shows that we cannot have forbidden subgraph characterization of d.e.i. graphs of diameter \( k \) for all \( k \geq 2 \).

4. **Cartesian Product Method.**

The product of two graphs \( G \times H \) is the graph has the vertex set \( V(G) \times V(H) \) and edges of the form \((a, a'), (b, b')\) such that \((a, b), (a', b')\) in \( E(G \times H) \) if and only if \( a = a' \) and \( b = b' \) in \( E(H) \) or \((a, a')\) in \( E(G) \) and \( b = b' \).

**THEOREM 7.** If \( G \) and \( H \) with \( D(G) \geq 2 \) and \( D(H) \geq 2 \) then \( G \times H \) is d.e.i. with diameter \( m+n \).

The conditions that both of the diameters of \( G \) and \( H \) greater than 1 are necessary. For the following results show that if \( G \) is complete, then \( G \times H \) need not be in DEI.

**THEOREM 8.** (a) The grid graph \( P_n \times P_m \) is diameter e-invariant if and only if \( \min \{ n, m \} \geq 3 \).

(b) The graph \( C_n \times P_m \) is d.e.i. if and only if \( n \neq 3 \) and \( m \geq 2 \).

**Proof.** (a) If \( n = 2 \) then we have a ladder \( P_2 \times P_m \) whose diameter is \( m \) (Figure 4(a)). However, \( P_2 \times P_m \setminus e \) has diameter \( m + 1 \) (Figure 4(b)). The other part follow from Theorem 7.

![Figure 4(a) D(P_2 \times P_m) = m.](image)

![Figure 4(b) d(a, b) = m + 1.](image)

(b) If \( n = 3 \) then we see that \( G = C_3 \times P_m \) has diameter \( m \) and \( G \setminus e \) has diameter \( m + 1 \) (Fig. 5).

![Figure 5. m = 2.](image)