We have immediately:

**Theorem 6.** The functor \( g : \text{Graph} \rightarrow \text{Gpd} \) is faithful, and for each \( G \) in \( \text{Graph} \), the group of automorphisms of \( G \) is isomorphic to the group of automorphisms of its associated graph algebra \( g(G) \).

A category \( C \) is said to be **group universal** if for each group \( H \) there exists an object \( X \) in \( C \) such that \( \text{Aut}_C(X) = H \).

Frucht [2] and Sabidussi [12] showed that the category of graphs is group universal, and Hedrlin and Mendelsohn [8] considered other group universal categories of graphs.

A 2-groupoid \( <S,*> \) is said to be **simple** if its lattice of congruences \( \text{Con}(S) \) is isomorphic to the two-element lattice, i.e. its only congruences are the equality relation and \( S \times S \).

McNulty and Shallon [11] showed the following:

**Theorem 7.** The graph algebra \( <g(G);*> \) of a graph \( G \) is simple if and only if

(i) \( G \) is connected and

(ii) for any \( x \neq y \) in \( V(G) \), \( N_G(x) \neq N_G(y) \).

**Theorem 8.** Every graph algebra whose base graph is connected and loopless is a subalgebra of a simple DT-graph algebra.

**Proof.** We assume that the graph \( G \) is not trivial. Let \( G = (V,E) \) and let \( V^* \) be a set with the same cardinality as \( V \), with the bijection \( x \rightarrow x^* \). Also, let \( c \notin V \cup V^* \). We construct \( S(G) = (G \circ K_1 \text{ (with loop)}) + K_1 \text{ (without loop)} \), where \( \circ \) is the corona construction of Frucht and Harary [3].

We see that \( V(S(G)) = V \cup V^* \cup \{c\} \) and \( E(S(G)) = E(G) \cup \{(x,x^*) : x \in V\} \cup \{(x^*,x^*) : x^* \in V^*\} \cup \{(c,x) : x \in V \cup V^*\} \), and also \( N_S(G)(x) = \{c,x^*\} \cup N_G(x) \) for \( x \in V(G) \), \( N_S(G)(c) = V \cup V^* \), and \( N_S(G)(x^*) = \{x,x^*,c\} \).