edge-graceful then for some \( k \geq 1 \)

\[
[ q(q + 1) - (2n + 1)(4n + 1) ] / 2(2n + 1) = k.
\]

Thus \( q(q + 1) = (2n + 1)(4n + 2k + 1) \).

Note the left hand side of the equality is an even number and the right hand side is an odd number, which is a contradiction.

Lo [1] conjectured that \( K_{4n} \) is edge-graceful for every \( k \geq 1 \). Let \( p \) be an integer not of the form \( 4k + 2 \). Now, we introduce an edge labeling on \( K_p \) as follows: (Fig. 3)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>(p-2)</th>
<th>(p-1)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(p-1)(p-2)/2</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>5</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>p(p-1)/2</td>
</tr>
</tbody>
</table>

**Fig. 3**

For simplicity, we shall denote \( g(e(i,j)) \) by \( e[i,j] \). By numbering the edges in the manner shown above it can be seen that if \( e[i,j] \) is the label from edge \( i \) to edge \( j \), then for \( K_p \) we have

\[
e[1,p] = 1
\]
\[
e[1, p - 1] = 2 = e[1,p] + 1
\]
\[
e[1, p - 2] = 4 = e[1, p - 1] + 2
\]
\[
e[1, p - 3] = 7 = e[1, p - 2] + 3
\]
\[
e[1, p - 4] = 11 = e[1, p - 3] + 4
\]

and so on. From this it can be shown that

\[
e[1,j] = (p - j)(p - j + 1)/2 + 1, \text{ for } 2 \leq j \leq p
\]

Similarly, it can be seen that