It can be easily checked that $f$ is a $S$-labeling.

![Figure 8](image)

Figure 8. A $S$-labeling of $St(m, n, t)$, where $m$, $n$, and $t$ are even.

Example 3.3. A $S$-labeling of $St(4, 2, 4)$. (Fig. 9)

![Figure 9](image)

Figure 9. A $S$-labeling of $St(4, 2, 4)$.

Theorem 4. A 3-star $St(m, n, t)$ is not Skolem-graceful if $m$, $n$, and $t$ are all odd.

Proof. Suppose $f$ is a $S$-labeling of $St(m, n, t)$ with $|V| = k + 3$ and $|E| = k$, where $k = m + n + t$. We have $|V_e| = |V_o| = (k+3)/2$, because $k$ is odd. Hence, it is easy to verify that $|V_e| = \per(k/2+1) + 2$ and $|V_o| = \per k/2| + 1$ and, $|E_e| = \per k/2|_e$ and $|E_o| = \per k/2|_o$. We use counting argument to prove the theorem. We consider the following four cases:

Case 1. $\{ f(A), f(B), f(C) \} = \{ \text{odd, even, even} \}$. Without loss of generality, we can assume $f(A)$ is odd, and $f(B)$ and $f(C)$ are even, then $|E_e| \leq |V_{mo}| + |V_{ne}| + |V_{te}|$. Assume $|V_{me}| = i$ and $|V_{ne}| = j$, then $|V_{me}| = m-i$ and $|V_{ne}| = n-j$. So, $|E_e| \geq (m-i) + (n-j) + \per k/2|_e$. Combining the two equations, we get $m-i+n-j+\per k/2|_e = \per k/2|_o$ and simplify it to get $m = 2i$. This is a contradiction, for $m$ is odd.

Case 2. $\{ f(A), f(B), f(C) \} = \{ \text{odd, odd, even} \}$. Without loss of generality, we can assume $f(A)$ and $f(B)$ are odd and $f(C)$ is even. Then $|E_e| = |V_{mo}| + |V_{no}| + |V_{te}|$. Assume $|V_{me}| = i$ and $|V_{ne}| = j$, then $|V_{mo}| = m-i$, $|V_{no}| = n-j$, and $|V_{te}| = \per(k/2+2) - i - j - 1$. So $|E_e| = (m-i) + (n-j) + \per(k/2+2) - i - j - 1$. Combining the two equations, we get $m-i+n-j+\per(k/2+2) - i - j - 1 = \per k/2|_o$. Simplify it, we get $m+n = 2(i+j) - 1$. This is a contradiction, because $m$ and $n$ are odd, their sum must be even.