Directions and rules. The exam will last 70 minutes; the last five minutes of class will be used for collecting the exams. No electronic devices of any kind will be allowed, with one exception: a music player that nobody else can hear, and whose controls you do not use during the exam (just put it on shuffle). Anything (else) with an off switch must be off. In particular, turn your cell phone off.

Show your work. Scoring: 6 problems, each worth 16 points, plus 4 free points = 100.

1. “Eve of Naharon” is the skeleton of a 25 to 30 years old female found in the Naharon section of the underwater cave Sistema Naranjal in Mexico near the town of Tulum, around 80 miles south west of Cancún. Radiocarbon analysis showed that 19.3% of the original amount of carbon 14 remained. How old is the skeleton? Your answer should be accurate to within one percent.

To do this problem without a calculator you can make use of some of these values:

\[
\ln(0.193) = -1.64506509 \\
\ln 2 = -\ln(1/2) = 0.693 \\
\log_2(0.193) = -2.37332725 \\
\text{Half life of } C^{14} = 5730 \text{ years}
\]
2. Consider a cylindrical storage tank, ten meters high. Use differentials to determine how accurately we must measure the interior diameter of the tank, in order to calculate the tank’s volume to within 1% of its true value?

3. (a) sketch the graph of \( \cosh x \)

(b) sketch the graph of \( \sinh x \)

(c) sketch the graph of \( \tanh x \)
4. Differentiate each of the following with respect to $x$, expressing the answer using hyperbolic trig functions (i.e., not using $e^x$).
   
   (a) $\cosh(\sqrt{x})$

   (b) $\tanh(1/x)$

5. Use the definitions of sinh and cosh to verify the identity.
   
   $\cosh 2x = \cosh^2 x + \sinh^2 x$
6. A girl flies a kite at a height of 300 feet. The wind carries the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out more string when she has already let out 500 feet of string? Assume that the string forms a straight line from the girl to the kite.

(a) Introduce two variables (in addition to $t$ for time) and draw a picture showing what the variables mean.

(b) Write an equation or equations connecting your variables, true for all the time the kite is flying as described.

(c) Write one or more additional equations that are only good at one special time.

(d) Finish solving the problem, to get the answer in radians per hour.