Directions and rules. The exam will last 2 hours and 15 minutes. No electronic devices of any kind will be allowed, with one exception: a music player that nobody else can hear, and whose controls you do not use during the exam (just put it on shuffle). Anything (else) with an off switch must be off. In particular, turn your cell phone off. There are 16 questions on the exam, counting equally.

1. Below is the graph of a function $f(x)$. From the graph, read off the value (if any) of the following limits:

$$
\lim_{x \to 1^-} f(x) = \\
\lim_{x \to 1^+} f(x) = \\
\lim_{x \to 0^-} f(x) = \\
\lim_{x \to 0^+} f(x) = \\
\lim_{x \to 1^-} f(x) = \\
\lim_{x \to 1^+} f(x) = 
$$
2. Big Bird rides the roller coaster at the Santa Cruz Boardwalk. After he boards the train, the train is lifted up the first big slope by a chain under the track. The chain carries the train up at a constant speed. When the train reaches the top, it is released to go down the hill under the force of gravity. Assume that the hill has a long straight slope, only rounding off very near the bottom to a flat section of track. We will only consider the up-and-down part of the ride, not the part after coming down the straight slope. Also we will only consider his height and his vertical velocity and acceleration. Assume that the boarding platform and the bottom end of the straight track are at the same height, and that the train is very short, so we can ignore the short time when the front of it is over the top but the back is still coming up.

Make three graphs showing his height above the ground, his velocity, and acceleration as functions of time. Make the graphs one above the other so the time scales line up.
3. Evaluate the following limits.

\[
\lim_{x \to 2} (x^4 + 3x + 1)
\]

\[
\lim_{x \to 2} \frac{x^2 - 4}{x - 2}
\]

\[
\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}
\]

\[
\lim_{t \to \infty} \left( \frac{t^3}{e^3 - 1} - 1 \right)
\]

4. (a) Fill in the blanks to make the following correct:

To prove (from the definition of limit) that \(3\sqrt{|x|}\) approaches 0 as \(x\) approaches zero:

Let \(\epsilon > 0\) be given.

Then choose \(\delta = \) ___________.

Then we verify that if ___________ then ___________.

(b) Draw a picture showing part of the graph of \(y = 3\sqrt{|x|}\) and labeling \(\epsilon\) and \(\delta\).
5. For each of the following functions, if the function is not continuous at every \( x \), at which \( x \) is it not continuous? If it is continuous everywhere then just write “continuous everywhere.”

\[
\begin{align*}
x - 1 \\
\frac{x}{x^2 - 1} \\
3 \sqrt{|x|} \\
tan x \\
e^{\sin x} \\
x \cos \frac{1}{\sqrt{x}}
\end{align*}
\]

6. Evaluate the following limits.

\[
\lim_{x \to \infty} \left( 4 \sqrt{x} - \sqrt{x} \right)
\]

\[
\lim_{t \to \infty} \left( \frac{t^3}{t - 1} - t^2 \right)
\]
7. Below is the graph of a function. Sketch the graph of its derivative on the second, blank graph given.
8. Evaluate the following derivatives

\[
\frac{d}{dx}(x^{49} + 2x + 1)
\]

\[
\frac{d}{dx}((x^{32} + 3x^3)(\sin x + 1))
\]

\[
\frac{d}{dx}\sqrt{x}e^x
\]

\[
\frac{d}{dx}\frac{1}{x^3 + 1}
\]

\[
\frac{d}{dx}(x^3 \sin x)
\]
9. Evaluate the following derivatives
\[
\frac{d}{dx} \ln(\ln x)
\]
\[
\frac{d}{dx} \sin(x^8)
\]
\[
\frac{d}{dx} (\sin 2x \cos x)
\]

10. Suppose \( x^3 + \tan y = 1 \). Calculate \( \frac{dy}{dx} \) using implicit differentiation, getting an answer that involves both \( x \) and \( y \).
11. “Eve of Naharon” is a skeleton of a young female found in an underwater cave in Mexico. Radiocarbon dating done in 2008 showed that only 19.3% of Eve’s original carbon 14 remains. What was Eve’s death date, accurate to the nearest ten years?

Here are some useful numbers: Half-life of carbon 14 is 5730 years; \( \ln(2) = 0.693 \); \( \log_2(0.193) = -2.3733 \); \( \ln(0.193) = -1.645 \).

The only acceptable answer is an integer for the year of the death.
12. Two commercial airplanes are flying at 40,000 feet along straight-line courses that intersect at right angles. Plane $A$ is approaching the intersection point at a speed of 442 miles per hour. Plane $B$ is approaching the intersection at 481 miles per hour. At what rate is the distance between the planes changing when $A$ is 5 miles from the intersection point and $B$ is 12 miles from the intersection point?

(a) Draw a diagram of this situation. Define three variables (other than $t$ for time) and indicate their meaning on the diagram.

(b) What, in symbols, is the unknown quantity?

(c) Write an equation connecting the variables that is good all the time the planes are moving as described.

(d) Finish solving the problem.
13. At the Monterey Bay Aquarium, there is a system of large pipes to supply sea water to the big tanks. The pipes tend to become partly clogged with plant material, which restricts the flow of water, so from time to time they clean the pipes (exactly how is interesting but not relevant to this problem). A similar problem arises in oil pipelines, which also need to be cleaned out from time to time. The equation relating flow and radius is \( V = kr^4 \), where \( r \) the radius of the pipe, \( k \) is a constant, and \( V \) is the volume of water that can flow in one second past a given point. Suppose that due to stuff accumulating on the inside of the pipe, the radius decreases by 4\%. By what percent will the flow decrease?

14. Find the numerical value of \( \cosh(\ln 10) \). Show your work, and be sure that your answer is a number.
15. Consider the function $f(x) = x^3 - 4x^2 + 5x + 1$. You can show your work below in the space provided for it, but please put your answers right after the questions where they will be easy to find. (The question will be graded as a whole; do not assume that all the parts have equal value.)

(a) For which $x$ is $f(x)$ increasing?
(b) For which $x$ is $f(x)$ decreasing?
(c) Find the values of $x$ and $f(x)$ at the local minima of $f$.
(d) Find the values of $x$ and $f(x)$ at the local maxima of $f$.
(e) Find the points of inflection of $f$ on the given interval.
(f) Where is $f$ concave up?
(g) Concave down?
(h) Sketch the graph of $f$ showing all the minima, maxima, and points of inflection.

The rest of the page is for your work.
16. Evaluate the following limits:

\[
\lim_{x \to 0} \frac{x^3}{\ln x}
\]

\[
\lim_{x \to 0} \frac{\ln(\cos x)}{\sin x}
\]

\[
\lim_{x \to 0} \frac{\ln(x^2 + 10)}{\ln x}
\]