Cell phones, pagers, etc. should all be off! If such a device goes off, you might be officially finished with your test. No tools (calculators, computers, sliderules, screwdrivers, friends, books, or notes) should be used for the test, other than a pen/pencil, your wits and knowledge. You should try to answer all questions for the rote exams. Good luck.

You may use the Master Theorem. It follows, in a somewhat over-simplified format. (Restrictions for case 3 have been dropped, they won’t be needed for the first problem of this test.) Also note, I have now moved the $\epsilon$ term to be within the parenthesis.

For $T[n] = aT[n/b] + f(n)$, then

1. If $f(n) = O(n \log_b (a-\epsilon))$ for some constant $\epsilon > 0$, then $T[n] = \Theta(n \log_b a)$.
2. If $f(n) = \Theta(n \log_b a)$, then $T[n] = \Theta(f(n) \lg n)$.
3. If $f(n) = \Omega(n \log_b (a+\epsilon))$ for some constant $\epsilon > 0$, then $T[n] = \Theta(f(n))$.

1. (10 points) Prove $\Theta(\ )$ notation for the following recurrence relation: SOME RECURRENCE HERE. THE MASTER THEOREM WILL APPLY.

Fill in the blanks:

This recurrence relation falls into the format of the Master Theorem, with $a =$, $b =$, and $f(n) =$. This will fall into case . If we are in case 1 or case 3, the largest possible $\epsilon$ value we can use is . (If we are in case 2, fill $\infty$ into the previous blank.) Work to show that this is the correct case: __________________________.

For this example, the Master Theorem tells us that the recurrence relation grows like $\Theta($_________).  

2. (10 points) For SOME RECURRENCE RELATION HERE, prove SOME RELATION HERE. THE SUBSTITUTION METHOD WOULD BE A GOOD CHOICE. THE MASTER THEOREM? NOT SO MUCH.

First, we inductively assume that for $x < n$, _______________________ holds. Using that inductive hypothesis and our given recurrence relation, we know that ________________________, and we want to prove that _______________________.

The relation we are trying to prove holds for values $C =$ and $n_0 =$, as is shown by the math below. (Show your math below.)

3. (10 points) Consider the following loop. You may assume that $n > 0$. I WILL GIVE YOU A LOOP HERE.

I WILL GIVE YOU A LOOP INVARIANT HERE.

Prove the loop invariant holds.

**Initialization:** The first time we get to the location, the program variables have the values ____________________, and the invariant holds because ____________________.

**Maintenance:** Assuming that the invariant holds for one iteration, one iteration later, the program variables have the values ____________________, and the invariant holds because ____________________.

**Termination:** The loop exits when the program variable(s) take the value(s) ____________________.

What do the invariant and the termination conditions prove about the loop when it exits?
4. (10 points) You will get a question involving at least one of: 23-trees, top-down 234 trees, or bottom up 234 trees. You should know how to insert into each of them, and know the differences between them.

5. (10 points) List the comparisons made, in order, for (bottom-up and/or top-down) mergesort, or quicksort and/or quickselect (to be specified) on the following list of numbers. LIST GOES HERE.

6. (10 points) I will give you some runtime question here. It will involve insertion sort. Given its runtime on some particular input, you will be asked what its expected runtime is on a different input, and perhaps on a different sized machine. You should know that insertion sort’s runtime is $\Theta(n + x)$ where $x$ is the number of inverted pairs in the starting array. (NOTE: FOR THE FINAL EXAM, YOU ARE EXPECTED TO KNOW THE RUNTIMES OF ALL BASIC ALGORITHMS WE HAVE SEEN.)

7. (10 points) You are in the middle of running heapsort, and are in the midst of deleting values from the heap. The array of values is given below. Show the array after 2 more values have been deleted. List all comparisons made between elements.

8. (10 points) You should know what a clique, independent set, and vertex cover are, well enough to answer questions about them on simple graphs. You should understand the definition of NP, in the context of those topics.