CS 154	**April 16, 2009**
Formal Languages & Computability	**Midterm 2 Format (1 page(s) total)**

Cell phones, pagers, etc. should all be off! No tools (calculators, computers, sliderules, screwdrivers, friends, books, or notes) should be used for the test, only a pen/pencil, your wits and knowledge.

For this exam, you may leave up to 2 of the first 4 questions blank, and up to 2 of the last 4 questions blank. There are only 8 questions. Remember how tests are scored: a blank answer is worth 30% credit. (You may also draw a large X through your scratchwork to indicate that you would prefer to take the 30%.) Spend your time working on questions which you think you can answer correctly.

Some useful facts:

- Regular language closure laws seen: if L_x and L_y are regular languages, than so are $L_x \cap L_y, L_x \cup L_y, L_x \cdot L_y, L_x^*, \overline{L_x}, L_x^R$, $L_x \ominus L_y$, $L_x - L_y$, and $h(L_x)$ for homomorphism $h(\ )$.
- The regular language pumping lemma: If L is a regular language, there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as $w = xyz$ with $|xy| \leq m$ and $|y| \geq 1$ such that $w_i = xy^iz$ is also in L for all non-negative integers i.
- You may use the fact that the following languages **are not** regular, but **are** context-free, without proof: $\{a^nb^n : n \geq 0\}$ and $\{ww^R : w \in \{a,b\}^*\}$.
- Context-free language closure laws we have seen: if L_x and L_y are context-free languages, than so are $L_x \cup L_y$, $L_x \cdot L_y$, L_x^*, L_x^R, $h(L_x)$ for homomorphism $h(\ )$, and $L_x \cap L_z$ if L_z is regular.
- Context-Free Language Pumping Lemma: If L is a context-free language, there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as $w = uvxyz$ with $|vxy| \leq m$ and $|vy| \geq 1$ such that $w_i = uv^ixy^iz$ is also in L for all non-negative integers i.
- You may use the fact that the following languages are **not** context-free, without proof: $\{a^nb^nc^n : n \geq 0\}$, $\{ww : w \in \{a,b\}^*\}$, $\{a^nb^ma^nb^m : n \geq 0\}$, and $\{a^n : n \text{ is a prime number}\}$.

1. Given a grammar, give a parse tree *and* leftmost derivation for the following string.
2. Prove that the following language is context free, either by giving a grammar for it, or by giving a PDA for it. (Give PDA in diagram format.)
3. Put the following Context Free Grammar into Chomsky Normal Form, after eliminating useless production rules. Show work.
4. For a Context Free Grammar, give a NPDA for the same language. Assume the PDA stack starts with a Z on it. Your PDA should have only 3 states, and be given via diagram, rather than a table. Follow rules we have seen.
5. Prove that the following language is not context free, using closure rules.
6. Prove that the following language is not context free, using the context free pumping lemma.
7. Given a Context Free Grammar, give the shortest pumpable string you can find for the language generated by that grammar, and show how to break the string into its u, v, x, y, z parts to be pumped.
8. Given the following 3 languages, indicate whether or not each language is regular, context free but not regular, or not context free. You do *not* need to prove your answer. 3 points per correct answer, 1 bonus for all 3 correct.