CS 154 (Taylor)	**Spring 2008**
Formal Languages & Computability	**Review Questions for Midterm 2 (1 page(s) total)**

Note: you may skip up to 3 (10 point) questions, taking a 30% default for each of those questions. Also note that this exam is quite different than those of past years, more similar to my first midterm than my second, due to differences in coverage by the lecturer. All specific languages, grammars, and automata given may be changed for the exam, these are just examples.

Some useful facts:

- Closure laws we have seen: if L_x and L_y are regular languages, than so are $L_x \cap L_y, L_x \cup L_y, L_x \cdot L_y, L_x^*, \overline{L_x}$, and L_x^R are regular.
- The regular language pumping lemma: If L is a regular language, there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as $w = xyz$ with $|xy| \leq m$ and $|y| \geq 1$ such that $w_i = xy^iz$ is also in L for all non-negative integers i.
- We have seen that the following languages are **not** regular. $\{a^nb^n : n \geq 0\}, \{ww^R : w \in \Sigma^*\}, \{a^n : n \text{ is a prime number}\}$.
- You may use the fact that the following languages **are not** regular, but **are** context-free, without proof: $\{a^nb^n : n \geq 0\}$, and $\{ww^R : w \in \{a, b\}^*\}$
- Context-free language closure laws we have seen: if L_x and L_y are context-free languages, than so are $L_x \cup L_y$, $L_x \cdot L_y$, L_x^*. Additionally, so are L_x^R and $L_x \cap L_z$ if L_z is regular.

1. (10 points) Following the procedure we have seen for building an NFA from a regular expression, give an NFA for $(a \cdot b) + (c^*)$. **Do not improvise.** Show steps for partial credit.
2. Consider the following NFA M_8:
   Give a right-linear grammar for M_8 following the rules we have seen in class and the text
   or
   Give an NFA for the following right-linear grammar.
3. (10 points) Prove that some language (such as the complement of $\{0^n1^{2n}\}$) is not regular.
   or: <u>*Use closure rules*</u> to show that the following language is not regular:
   $L_{17} = \{w : w \in \{a, b, c\}^*$ and the number of a's equals the number of b's plus the number of c's.$\}$
   So, *abcaaacc* is in L_{17}, but *abcabc* and *aabbcc* are not. (Note, this question may be replaced by a closure question on context free languages, with appropriate information given to you to answer it.)
4. (20 points total) For each of the following languages, indicate whether or not it is regular, or context-free but not regular. Justify your answers: for regular languages, give a regular expression, DFA, NFA, or right-linear grammar. If not regular, prove it, and show it is context free with a context-free grammar push down automata. (Most credit comes from the proof, not the regular/not-regular answer.) (Two questions will be given here, each worth 10 points, such as $L_{13} = \{wawa : w \in \{b\}^*\}$, $L_{14} = \{bwbw : w \in \{b\}^*\}$, or $L_{15} = \{a^ib^j : i \leq j\}$
5. (10 points) Prove that, if L_a and L_b are regular, than so is SOME LANGUAGE GOES HERE.
6. (10 points) Prove that the language $L_8 = \{ba^nb^n : n \geq 0\}$ is not regular using the regular language pumping lemma.
7. (10 points) Prove that the following language is context-free: $L_9 = \{c^ma^nb^nc^m\}$
8. (10 points) Consider the following grammar:
   $S \rightarrow ABA|BC$
   $A \rightarrow aAb|a$
   $B \rightarrow bB|bC$
   $C \rightarrow dS|c$
   Give a parse tree and leftmost derivation for the string *bbdbccc*
9. (10 points) For each of the following languages, indicate whether or not the language is regular, context-free but not regular, or not context-free. You do *not* need to prove your answer. 3 points per correct answer, 1 bonus for all 3 right.
   (a) $\{a^nb^mc^ma^lb^lc^n : n, l \geq 0, m \geq 1\}$
   (b) $\{a^na^mb^nb^m : n, m \geq 0\}$
   (c) $\{a^nb^nc^n : n \geq 0\} \cup \{a^nb^mc^l : n, m, l \geq 0\}$