### Classic Crypto

Classic Crypto

### Overview

#### We briefly consider the following classic (pen and paper) ciphers

- Transposition ciphers
- Substitution ciphers
- o One-time pad
- o Codebook
- These were all chosen for a reason
  - We see same principles in modern ciphers

### **Transposition Ciphers**

- In transposition ciphers, we transpose (scramble) the plaintext letters
  - The scrambled text is the ciphertext
  - The transposition is the key
- Corresponds to Shannon's principle of diffusion (more about this later)
  - This idea is widely used in modern ciphers





□ Spartans, circa 500 BC

Wind strip of leather around a rod

Write message across the rod

Η Ε Μ Н A Ε 0 S M Η E W S S R U Α Ι A D 0 : Т A Κ 0 F М

When unwrapped, letters are scrambled TSATAHCLONEORTYTMUATIESLHMTS...







Suppose Alice and Bob use Scytale to encrypt a message

• What is the key?

• How hard is it for Trudy to break without key?

- Suppose many different rod diameters are available to Alice and Bob...
  - How hard is it for Trudy to break a message?
  - Can Trudy attack messages automatically—without manually examining each **putative** decrypt?

### Columnar Transposition

- Put plaintext into rows of matrix then read ciphertext out of columns
- $\Box$  For example, suppose matrix is 3 x 4
  - o Plaintext: SEETHELIGHT

- Ciphertext: SHGEEHELTTIX
- Same effect as Scytale
  - What is the key?

### Keyword Columnar Transposition

#### □ For example

• Plaintext: CRYPTOISFUN

• Matrix 3 x 4 and keyword MATH

М	Α	Т	Η	_
C	R	Y	Ρ	1
Т	0	Ι	S	
F	U	Ν	X	

- Ciphertext: ROUPSXCTFYIN
- What is the key?
- How many keys are there?

### Keyword Columnar Transposition

- How can Trudy cryptanalyze this cipher?
- Consider the ciphertext

VOESA IVENE MRTNL EANGE WTNIM HTMLL ADLTR NISHO DWOEH

- □ Matrix is n x m for some n and m
- □ Since 45 letters,  $n \cdot m = 45$
- How many cases to try?
- □ How will Trudy know when she is correct?

### Keyword Columnar Transposition

#### The ciphertext is

■ If encryption matrix was 9 x 5, then...



Classic Crypto

### Cryptanalysis: Lesson I

#### Exhaustive key search

• Always an option for Trudy

If keyspace is too large, such an attack will not succeed in a reasonable time

• Or it will have a low probability of success

- □ A large keyspace is necessary for security
- But, large keyspace is not sufficient...

#### Plaintext: ATTACK AT DAWN

columns	0	1	2	Permute rows	columns	0	2	1
row O	A	Т	Т	and columns	row 2	Х	Т	A
row 1	A	С	K		row 4	W	Х	Ν
row 2	Х	A	Т	$ \rightarrow $	row O	А	Т	Т
row 3	Х	D	A		row 3	Х	A	D
row 4	W	Ν	Х		row 1	А	K	C

**Ciphertext:** XTAWXNATTXADAKC □ Key?

o 5 x 3 matrix, perms (2,4,0,3,1) and (0,2,1) Classic Crypto 11

- How can Trudy attack double transposition?
- Spse Trudy sees 45-letter ciphertext
- Then how many keys?
  - Size of matrix: 3 x 15, 15 x 3, 5 x 9, or 9 x 5
  - A lot of possible permutations!

 $5! \cdot 9! > 2^{25}$  and  $3! \cdot 15! > 2^{42}$ 

- $\square$  Size of keyspace is greater than  $2^{43}$
- Is there a shortcut attack?

- Shortcut attack on double transposition?
- Suppose ciphertext is

ILILWEAHREOMEESANNDDVEGMIERWEHVEMTOSTTAONNTNH

- Suppose Trudy guesses matrix is 9 x 5
- Then Trudy has:
- Now what?
- □ Try all perms? 5! · 9! > 2<sup>25</sup>
- □ Is there a better way?

column	0	1	2	3	4
row O	Ι	L	Ι	L	W
row 1	E	А	Η	R	Е
row 2	0	М	Е	Е	S
row 3	Α	Ν	Ν	D	D
row 4	V	Е	G	М	Ι
row 5	Ε	R	W	Е	Н
row 6	V	Е	М	Т	0
row 7	S	Т	Т	A	0
row 8	N	N	Т	N	Н

Shortcut attack on double transposition?

Trudy tries "columns first" strategy

column	0	1	2	3	4	
row O	I	L	I	L	W	
row 1	E	Α	Η	R	E	
row 2	0	М	E	E	S	Permute
row 3	A	Ν	Ν	D	D	columns
row 4	V	E	G	М	I	
row 5	E	R	W	E	Η	
row 6	V	E	М	Т	0	
row 7	S	Т	Т	Α	0	
row 8	N	N	Т	N	Η	

column	2	4	0	1	3
row O	Ι	W	Ι	L	L
row 1	Η	Е	Е	А	R
row 2	E	S	0	Μ	Ε
row 3	Ν	D	Α	Ν	D
row 4	G	Ι	V	Е	М
row 5	W	Η	Е	R	Ε
row 6	М	0	V	Е	Т
row 7	Т	0	S	Т	Α
row 8	Т	Η	Ν	Ν	Ν

#### Now what?

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### Cryptanalysis: Lesson II

#### Divide and conquer

- Trudy attacks part of the keyspace
- A great shortcut attack strategy
- Requires careful analysis of algorithm
- We will see this again and again in the attacks discussed later
- Of course, cryptographers try to prevent divide and conquer attacks

### Substitution Ciphers

- In substitution ciphers, we replace the plaintext letters with other letters
  - The resulting text is the ciphertext
  - The substitution rule is the key
- Corresponds to Shannon's principle of confusion (more on this later)
  - This idea is used in modern ciphers

### Ceasar's Cipher

Plaintext:

FOURSCOREANDSEVENYEARSAGO



### **Ciphertext**:

IRXUVFRUHDAGVHYHABHDUVDIR

More succinctly, key is "shift by 3"

### Ceasar's Cipher

## Trudy loves the Ceasar's cipher... Suppose ciphertext is VSRQJHEREVTXDUHSDQWU

PlaintextabcdefghijklmnopqrstuvwxyzCiphertextDEFGHIJKLMNOPQRSTUVWXYZABC

# Then plaintext is SPONGEBOBSQUAREPANTS

### Simple Substitution

- Caesar's cipher is trivial if we adhere to Kerckhoffs' Principle
- We want a substitution cipher with lots of keys
- What to do?
- Generalization of Caesar's cipher...

### Simple Substitution

Key is some permutation of letters
 Need not be a shift

#### □ For example

Plaintext a b c d e f g h i j k l m n o p q r s t u v w x y z Ciphertext J I C A X S E Y V D K W B Q T Z R H F M P N U L G O

Then 26! > 2<sup>88</sup> possible keys
 That's lots of keys!

### Cryptanalysis of Simple Substitution

Trudy know a simple substitution is used Can she find the key given ciphertext: PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVW LXTOXBTFXQWAXBVCXQWAXFQJVWLEQNTOZQGGQLFXQ WAKVWLXQWAEBIPBFXFQVXGTVJVWLBTPQWAEBFPBFH CVLXBQUFEVWLXGDPEQVPQGVPPBFTIXPFHXZHVFAGF **OTHFEFBQUFTDHZBQPOTHXTYFTODXQHFTDPTOGHFQP** BQWAQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFWPFHP **BFIPBQWKFABVYYDZBOTHPBQPQJTQOTOGHFQAPBFEQ** JHDXXQVAVXEBQPEFZBVF0JIWFFACFCCFHQWAUVWFL **QHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFHFQAI** TIXPFHXAFQHEFZQWGFLVWPTOFFA

## Cryptanalysis of Simple Substitution

- □ Trudy cannot try all 2<sup>88</sup> possible keys
- Can she be more clever?
- Statistics!

#### English letter frequency counts:



## Cryptanalysis of Simple Substitution

#### Ciphertext:

PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTF XQWAXBVCXQWAXFQJVWLEQNTOZQGGQLFXQWAKVWLXQWAEBIPBF XFQVXGTVJVWLBTPQWAEBFPBFHCVLXBQUFEVWLXGDPEQVPQGVP PBFTIXPFHXZHVFAGFOTHFEFBQUFTDHZBQPOTHXTYFTODXQHFT DPTOGHFQPBQWAQJJTODXQHFOQPWTBDHHIXQVAPBFZQHCFWPFH PBFIPBQWKFABVYYDZBOTHPBQPQJTQOTOGHFQAPBFEQJHDXXQV AVXEBQPEFZBVFOJIWFFACFCCFHQWAUVWFLQHGFXVAFXQHFUFH ILTTAVWAFFAWTEVOITDHFHFQAITIXPFHXAFQHEFZQWGFLVWPT OFFA

#### Ciphertext frequency counts:

A	В	С	D	E	F	G	Н	Ι	J	K	L	M	N	0	Ρ	Q	R	S	Т	U	V	W	X	У	Ζ
21	26	6	10	12	51	10	25	10	9	3	10	0	1	15	28	42	0	0	27	4	24	22	28	6	8

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### Cryptanalysis: Lesson III

### Statistical analysis

Statistics might reveal info about key
Ciphertext should appear random
But randomness is not easy

Difficult to define random (entropy)

Cryptographers work hard to prevent statistical attacks

### Poly-Alphabetic Substitution

Like a simple substitution, but permutation ("alphabet") changes
Often, a new alphabet for each letter
Very common in classic ciphers
Vigenere cipher is an example
Discuss Vigenere later in this section
Used in WWII-era cipher machines

### Affine Cipher

- Number the letters 0 thru 25
   A is 0, B is 1, C is 2, etc.
   Then offine cipher energy tion
- Then affine cipher encryption is defined by c<sub>i</sub> = ap<sub>i</sub> + b (mod 26)
  - ${\color{black}\bullet}$  Where  $p_i$  is the  $i^{th}$  plaintext letter
  - And a and b are constants
  - Require that gcd(a, 26) = 1 (why?)

### Affine Cipher

Encryption: c<sub>i</sub> = ap<sub>i</sub> + b (mod 26)
 Decryption: p<sub>i</sub> = a<sup>-1</sup>(c<sub>i</sub> - b) (mod 26)
 Keyspace size?
 Keyspace size is 26 · φ(26) = 312
 Too small to be practical

### Vigenere Cipher

□ Key is of the form  $K = (k_0, k_1, ..., k_{n-1})$ 

• Where each  $k_i \! \in \! \{0, 1, 2, \ldots, 25\}$ 

#### Encryption

$$c_i = p_i + k_{i \pmod{n}} \pmod{26}$$

#### Decryption

$$p_i = c_i - k_{i \pmod{n}} \pmod{26}$$

Nothing tricky here!

Just a repeating sequence of (shift by n) simple substitutions

### Vigenere Cipher

- □ For example, suppose key is MATH
  - That is, K = (12,0,19,7), since M is letter 12, and so on
- Plaintext: SECRETMESSAGE
- Ciphertext: EEVYQTFLESTNQ
- □ Encrypt:

S E C R E T M E S S G E Α 4 2 17 4 19 12 4 18 18 0 6 4 18 0 19 12 0 12 7 12 19 7 +12 7  $\mathbf{O}$ 19 4 21 24 5 11 16 19 18 19 13 16 (mod 26) 4 4 L Y Ε F Ε Ν Ε V Q Т S Т Q

### Vigenere Cipher

- Vigenere is just a series of k simple substitution ciphers
- Should be able to do k simple substitution attacks

• Provided enough ciphertext

But how to determine k (key length)?
 Index of coincidence...

Assume ciphertext is English letters
 Let n<sub>0</sub> be number of As, n<sub>1</sub> number of Bs, ..., n<sub>25</sub> number of Zs in ciphertext
 Let n = n<sub>0</sub> + n<sub>1</sub> + ... + n<sub>25</sub>
 Define index of coincidence

$$I = \frac{\binom{n_0}{2} + \binom{n_1}{2} + \dots + \binom{n_{25}}{2}}{\binom{n}{2}} = \frac{1}{n(n-1)} \sum_{i=0}^{25} n_i(n_i-1)$$

$$\Box \text{ What does this measure?}$$

- Gives the probability that 2 randomly selected letters are the same
- □ For plain English, prob. 2 letter are same:
  - $p_0^2 + p_1^2 + ... + p_{25}^2 \approx 0.065$ , where  $p_i$  is probability of i<sup>th</sup> letter
- □ Then for simple substitution,  $I \approx 0.065$
- □ For random letters, each  $p_i = 1/26$

• Then  $p_0^2 + p_1^2 + ... + p_{25}^2 \approx 0.03846$ 

□ Then I ≈ 0.03846 for poly-alphabetic substitution with a very long keyword

- How to use this to estimate length of keyword in Vigenere cipher?
- Suppose keyword is length k, message is length n
  - Ciphertext in matrix with k columns, n/k rows
- Select 2 letters from same columns

o Like selecting from simple substitution

Select 2 letters from different columns
 Like selecting random letters

- Suppose k columns and n/k rows
- Approximate number of matching pairs from same column, but 2 different rows:

$$0.065 \binom{\frac{n}{k}}{2} k = 0.065 \frac{1}{2} \left(\frac{n}{k}\right) \left(\frac{n}{k} - 1\right) k = 0.065 \left(\frac{n(n-k)}{2k}\right)$$

 Approximate number of matching pairs from 2 different columns, and any two rows:

$$0.03846 \binom{k}{2} \left(\frac{n}{k}\right)^2 = 0.03846 \frac{n^2(k-1)}{2k}$$

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Approximate index of coincidence by:

$$I \approx \frac{0.03846 \frac{n^2(k-1)}{2k} + 0.065 \left(\frac{n(n-k)}{2k}\right)}{\binom{n}{2}}$$
$$= \frac{0.03846n(k-1) + (0.065)(n-k)}{k(n-1)}$$

Solve for k to find:

 $k \approx \frac{0.02654n}{(0.065 - I) + n(I - 0.03846)}$ 

#### Use n and I (known from ciphertext) to approximate length of Vigenere keyword

Classic Crypto

### Index of Coincidence: Bottom Line

- A crypto breakthrough when invented
   By William F. Friedman in 1920s
- Useful against classical and WWIIera ciphers
- Incidence of coincidence is a wellknown statistical test
  - Many other statistical tests exists

### Hill Cipher

- Hill cipher is not related to small mountains
- Invented by Lester Hill in 1929
  - A pre-modern block cipher
- Idea is to create a substitution cipher with a large "alphabet"
- All else being equal (which it never is) cipher should be stronger than simple substitution

## Hill Cipher

- $\Box$  Plaintext,  $p_0$ ,  $p_1$ ,  $p_2$ , ...
- Each p<sub>i</sub> is block of n consecutive letters

• As a column vector

Let A be n x n invertible matrix, mod 26

- $\hfill\square$  Then ciphertext block  $c_i$  is given by
  - **o**  $c_i = A p_i \pmod{26}$
  - Decryption:  $p_i = A^{-1}c_i \pmod{26}$
- The matrix A is the key

### Hill Cipher Example

□ Let n = 2 and  $A = \begin{bmatrix} 22 & 13 \\ 11 & 5 \end{bmatrix}$ □ Plaintext

MEETMEHERE = (12, 4, 4, 19, 12, 4, 7, 4, 17, 4)

Then  $p_0 = \begin{bmatrix} 12\\ 4 \end{bmatrix}, p_1 = \begin{bmatrix} 4\\ 19 \end{bmatrix}, p_2 = \begin{bmatrix} 12\\ 4 \end{bmatrix}, p_3 = \begin{bmatrix} 7\\ 4 \end{bmatrix}, p_4 = \begin{bmatrix} 17\\ 4 \end{bmatrix}$ And  $c_0 = \begin{bmatrix} 4\\ 22 \end{bmatrix}, c_1 = \begin{bmatrix} 23\\ 9 \end{bmatrix}, c_2 = \begin{bmatrix} 4\\ 22 \end{bmatrix}, c_3 = \begin{bmatrix} 24\\ 19 \end{bmatrix}, c_4 = \begin{bmatrix} 10\\ 25 \end{bmatrix}$ Ciphertext: (4,22,23,9,4,22,24,19,10,25) = EWXJEWYTKZ

### Hill Cipher Cryptanalysis

- Trudy suspects Alice and Bob are using Hill cipher, with n x n matrix A
- SupposeTrudy knows n plaintext blocks
  - Plaintext blocks p<sub>0</sub>,p<sub>1</sub>,...,p<sub>n-1</sub>
  - Ciphertext blocks  $c_0, c_1, \dots, c_{n-1}$
- □ Let P be matrix with columns  $p_0, p_1, ..., p_{n-1}$
- □ Let C be matrix with columns  $c_0, c_1, ..., c_{n-1}$
- □ Then AP = C and  $A = CP^{-1}$  if  $P^{-1}$  exists

Cryptanalysis: Lesson IV Linear ciphers are weak • Since linear equations are easy to solve Strong cipher must have nonlinearity Linear components are useful • But cipher cannot be entirely linear Cryptanalyst try to approximate nonlinear parts with linear equations

### One-time Pad

- A provably secure cipher
- No other cipher we discuss is provably secure
- Why not use one-time pad for everything?
  - Impractical for most applications
  - But it does have its uses

### **One-time Pad Encryption**

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

	h	е	i	1	h	i	t	1	е	r	
Plaintext:	001	000	010	100	001	010	111	100	000	101	
Key:	111	101	110	101	111	100	000	101	110	000	
Ciphertext:	110	101	100	001	110	110	111	001	110	101	
	S	r	1	h	S	S	t	h	S	r	

### One-time Pad Decryption

e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

#### 

	S	r	1	h	S	S	t	h	S	r	
Ciphertext:	110	101	100	001	110	110	111	001	110	101	
Key:	111	101	110	101	111	100	000	101	110	000	
Plaintext:	001	000	010	100	001	010	111	100	000	101	
	h	е	i	1	h	i	t	1	е	r	

### One-time Pad

Double agent claims sender used "key": l h h s s t r S S r Ciphertext: 110 101 100 001 110 110 111 001 110 101 "key": 101 111 000 101 111 100 000 101 110 000"Plaintext": 010 100 100 001 011 010 111 100 000101 k i l l h i t l e r i=010 e=000 h=001 k=011 l=100 r=101 s=110 t=111

### One-time Pad

#### Sender is captured and claims the key is: l h s s t h S S r r Ciphertext: 110 101 100 001 110 110 111 001 110 101 "Key": 111 101 000 011 101 110 001 011 101 101 "Plaintext": 001 000 100 010 011 000 110 010 011 000elikesik h e e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111

### One-time Pad Summary

Provably secure, when used correctly

- Ciphertext provides no info about plaintext
- All plaintexts are equally likely
- o Pad must be random, used only once
- Pad is known only by sender and receiver
- Pad is same size as message
- No assurance of message integrity
- Why not distribute message the same way as the pad?

### Real-world One-time Pad

#### Project <u>VENONA</u>

- Soviet spy messages from U.S. in 1940's
- Nuclear espionage, etc.
- o Thousands of messaged
- □ Spy carried one-time pad into U.S.
- Spy used pad to encrypt secret messages
- Repeats within the "one-time" pads made cryptanalysis possible

### VENONA Decrypt (1944)

[C% Ruth] learned that her husband [v] was called up by the army but he was not sent to the front. He is a mechanical engineer and is now working at the ENORMOUS [ENORMOZ] [vi] plant in SANTA FE, New Mexico. [45 groups unrecoverable]

detain VOLOK [vii] who is working in a plant on ENORMOUS. He is a FELLOWCOUNTRYMAN [ZEMLYaK] [viii]. Yesterday he learned that they had dismissed him from his work. His active work in progressive organizations in the past was cause of his dismissal. In the FELLOWCOUNTRYMAN line LIBERAL is in touch with CHESTER [ix]. They meet once a month for the payment of dues. CHESTER is interested in whether we are satisfied with the collaboration and whether there are not any misunderstandings. He does not inquire about specific items of work [KONKRETNAYa RABOTA]. In as much as CHESTER knows about the role of LIBERAL's group we beg consent to ask C. through LIBERAL about leads from among people who are working on ENOURMOUS and in other technical fields.

- "Ruth" == Ruth Greenglass
- "Liberal" == Julius Rosenberg
- "Enormous" == the atomic bomb

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### Codebook Cipher

Literally, a book filled with "codes"

- More precisely, 2 codebooks, 1 for encryption and 1 for decryption
- Key is the codebook itself
- Security of cipher requires physical security for codebook
- Codebooks widely used thru WWII

### Codebook Cipher

#### Literally, a book filled with "codewords"

Zimmerman Telegram encrypted via codebook

Februar	13605
fest	13732
finanzielle	13850
folgender	13918
Frieden	17142
Friedenschluss	17149

Modern block ciphers are codebooks!
 More on this later...

Zimmerman Telegram

- One of most famous codebook ciphers ever
- Led to US entry in WWI
- Ciphertext shown here...

	bereal, which	are bereby age	and to	FIE	5	via Gali	veston	Dato	511
GE	MEXICO	CITY	0	10	1. M.			E.S.	
130	13042	13401	8501	115 3	528 41	6 172	14 649	1 113	10
18147	18222	21560	0 1024	7 1151	8 2367	7 1360	05 349	4 149	36
98092	5905	11311	10392	10371	0302	21290	5161	39695	
23571	17504	11269	9 1827	6 1810	1 0317	0228	17694	4473	
23284	22200	1945	2 2158	9 6789	3 5569	13918	8958	1213	;
1333	4725	4458	5905	17166	13851	4458	17149	14471	670
13850	12224	6929	14991	7382	15857	67893	14218	3647	,
5870	17553	67893	5870	5454	16102	15217	22801	17138	1
21001	17388	7440	23638	18222	6719	14331	15021	2384	5
3156	23552	22096	21604	4797	9497	22464	20855	4377	
23610	18140	22260	5905	13347	20420	39689	13732	20667	
3929	5275	18507	52262	1340	22049	13339	11265	22295	
10439	14814	4178	6992	8784	7632	7357 e	926 52	262 1	126
21100	21272	9346	9559	22464	15874	18502	18500	15857	
188	5376	7381	98092	16127	13486	9350	9220 7	6036 1	421
	2831	17920	11347	17142	11264	7667	7762	15099	911
5144					CONTRACTOR AND A DESCRIPTION	and a special second second second second	CONTRACTOR OF THE REAL PROPERTY OF	The second se	1. Mar. 10. Mar.

Zimmerman Telegram Decrypted

 British had recovered partial codebook
 Able to fill in

missing parts

By Much & Echleff Chilingint FROM 2nd from By Con 22,1957

FROM 2nd from London # 5747.

"We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of america neutral. In the event of this not succeeding, we make Mexico a proposal of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and arizona. The settlement in detail is left to you. You will inform the President of the above most . secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ZINNERMANN.

### Codebook Cipher

# Codebooks are susceptible to statistical analysis

Like simple substitution cipher, but lots of data required to attack a codebook
Historically, codebooks very popular
To extend useful life of a codebook, an additive was usually used

### Codebook Additive

#### Codebook additive is another book filled with "random" number

Sequence of additive numbers added to codeword to yield ciphertext



### Codebook Additive

- Usually, starting position in additive book selected at random by sender
- Starting additive position usually sent "in the clear" with the ciphertext
  - Part of the message indicator (MI)
  - Modern term: initialization vector (IV)
- Why does this extend the useful life of a codebook?

Cryptanalysis: Summary

- Exhaustive key search
- Divide and conquer
- Statistical analysis
- Exploit linearity
- Or any combination thereof (or anything else you can think of)
- All's fair in love and war...

o ... and cryptanalysis!