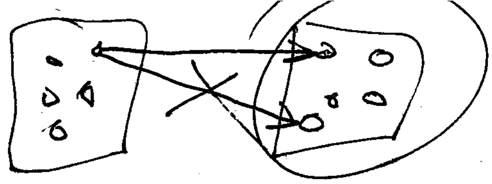


1) Let $N$ be Natural Numbers
Let $E$ be even numbers

Then the transformation
$f(x) = 2x$ for $x \in N$
~~bijection~~

$\{n \mid n = 2m \text{ for } m \in N\}$

Prove 1 to 1:

Assume there is some $k_1 \neq k_2$ such that $f(k_1) = f(k_2)$. Then

$$f(k_1) = 2k_1 = f(k_2) = 2k_2$$

divide by 2 $\qquad k_1 = k_2$

But we started w/ $k_1 \neq k_2$, so this is a contradiction. thus $f(x) = f(y)$ iff $x = y$.

Prove onto:
Show that for any $n \in$ Even Numbers there exists an $m \in N$ s.t. $n = 2m$

By definition, even numbers can be written as $2k$ for some $k \in N$.

2) $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

③ > Reflexive :

L1 is similar to itself. the difference is $\phi$.

> Symmetric :

Assume $L1 \sim L2$

(Thus $\underbrace{(L1-L2) \cup (L2-L1)}_{B}$ )

is finite.

Then $L2 \sim L1$ means:

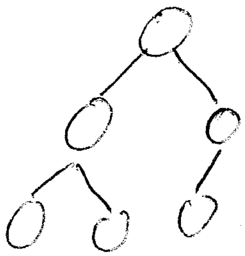
$A = (L2-L1) \cup (L1-L2)$ must be finite. This is true b/c the set A above is exactly the same set as B above.

> Transitive : If L1 is similar to L2 by the finite set A, and L2 is similar to L3 by B, then L1 is similar to L3 by $A \cup B$.

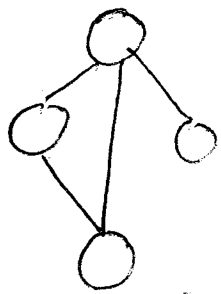
(which is a finite set)

Midterm I

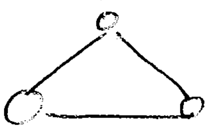
(4)



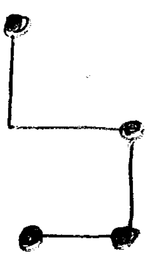
A tree ~~that is connected~~ has no cycle.



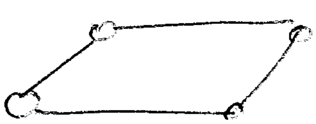
~~A tree that is~~ has cycle. (Ignore)



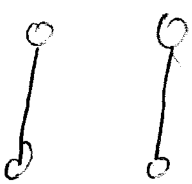
A graph with cycle.



A graph w/o cycle.



A graph that is connected



A graph not connected.

5 1st we argue that every binary boolean $f^n$ can be written using $\wedge, \vee, \neg$.

To see this consider the truth table for some binary Boolean function f:

| A | B | f |
|---|---|---|
| T | T | $y_1$ |
| T | F | $y_2$ |
| F | T | $y_3$ |
| F | F | $y_4$ |

Here $y_i$ are either T or F depending on the particular function f we are considering.

If $y_1$ is true we can construct a $f^n$ which checks for this row of the truth table by ANDing the literals of this row. The literal for A will be either A or $\neg A$ depending on whether the row had T or F for A. The literal for B is defined similarly. So if we had the row TF $y_2$ and $y_2$ is T, we could make a $f^n$ which checks for this as $(A \wedge \neg B)$. Now we do this process for each of the rows where f is T. Then we OR these rows together. So a table like:

| A | B | f |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

would become:

$$(A \wedge \neg B) \vee (\neg A \wedge B)$$

This completes the proof that every binary Boolean $f^n$ can be written using $\wedge, \vee, \neg$.

2nd To see we could use just NOR notice

$$\neg A \Leftrightarrow (A \text{ NOR } A)$$

$$(A \vee B) \Leftrightarrow ((A \text{ NOR } B) \text{ NOR } (A \text{ NOR } B))$$

$$A \wedge B) \Leftrightarrow \neg(\neg A \vee \neg B)$$

⑥ Want to prove by induction that

$$\frac{n!}{(n-k)!\,k!} = \frac{(n-1)!}{(n-k)!\,(k-1)!} + \frac{(n-1)!}{(n-1-k)!\,k!}$$

$\boxed{\text{Base case}}$ $n=2$, $0<k<n$ in this case means $k=1$

$$\frac{2!}{(2-1)!\,1!} = \frac{1!}{(2-1)!\,0!} + \frac{1!}{(2-1-1)!\,1!}$$

$$\frac{2}{1\cdot 1} = \frac{1}{1\cdot 1} + \frac{1}{1\cdot 1}$$

$2=2$ so base case holds

$\boxed{\text{Induction Step}}$

Assume for $n>2$, that for each $0<k<n$ we have

$$\frac{n!}{(n-k)!\,k!} = \frac{(n-1)!}{(n-k)!\,(k-1)!} + \frac{(n-1)!}{(n-1-k)!\,k!}$$

Consider the equation for $m=n+1$, $0<k<m$.

$$\frac{m!}{(m-k)!\,k!} = \frac{(n+1)!}{((n+1)-k)!\,k!} = \frac{(n+1)\left[\ n!\ \right]}{((n+1)-k)\left[(n-k)!\,k!\right]}$$

using the induction hypothesis on the circled part

$$= \frac{n+1}{(n+1)-k}\left[\frac{(n-1)!}{(n-k)!\,(k-1)!} + \frac{(n-1)!}{(n-1-k)!\,k!}\right]$$

$$= \frac{n(n-1)! + (n-1)!}{(n+1-k)!\,(k-1)!} + \frac{n(n-1)! + (n-1)!}{(n+1-k)\,(n-1-k)!\,k!}$$

$$= \frac{n!}{(n+1-k)!\,(k-1)!} + \frac{k(n-1)! + (n-k)\left(n! + (n-1)!\right)}{(n+1-k)!\ k!}$$

made common denominator

$$= \frac{n!}{(n+1-k)!\,(k-1)!} + \frac{(n-k)n! + \underbrace{(n\ \ )(n-1)!}_{=n!}}{(n+1-k)!\,k!}$$

$$= \frac{n!}{(n+1-k)!\,(k-1)!} + \frac{(n-k+1)\,n!}{(n+1-k)!\,k!}$$

cancel $n-k+1$

$$= \frac{n!}{(n+1-k)!\,(k-1)!} + \frac{n!}{(n-k)!\,k!}$$

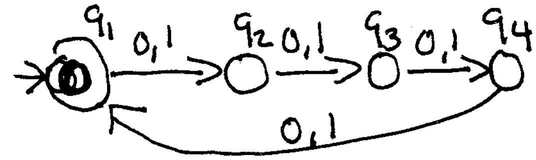
$$= \frac{(m-1)!}{(m-k)!\,(k-1)!} + \frac{(m-1)!}{(m-1+k)!\,k!}$$

so induction holds. QED

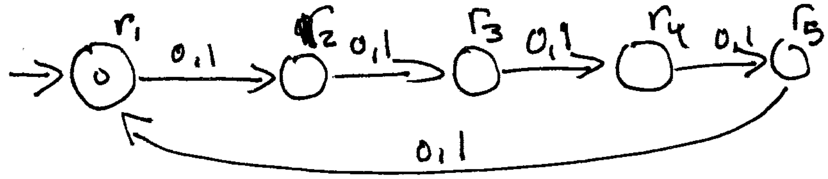
(7) (1st) Make machines $M, M'$ for $L = \{w \mid |w| = 4k$ for some $k \in \mathbb{N}\}$

$L' = \{w \mid |w| = 5k$ for some $k \in \mathbb{N}\}$

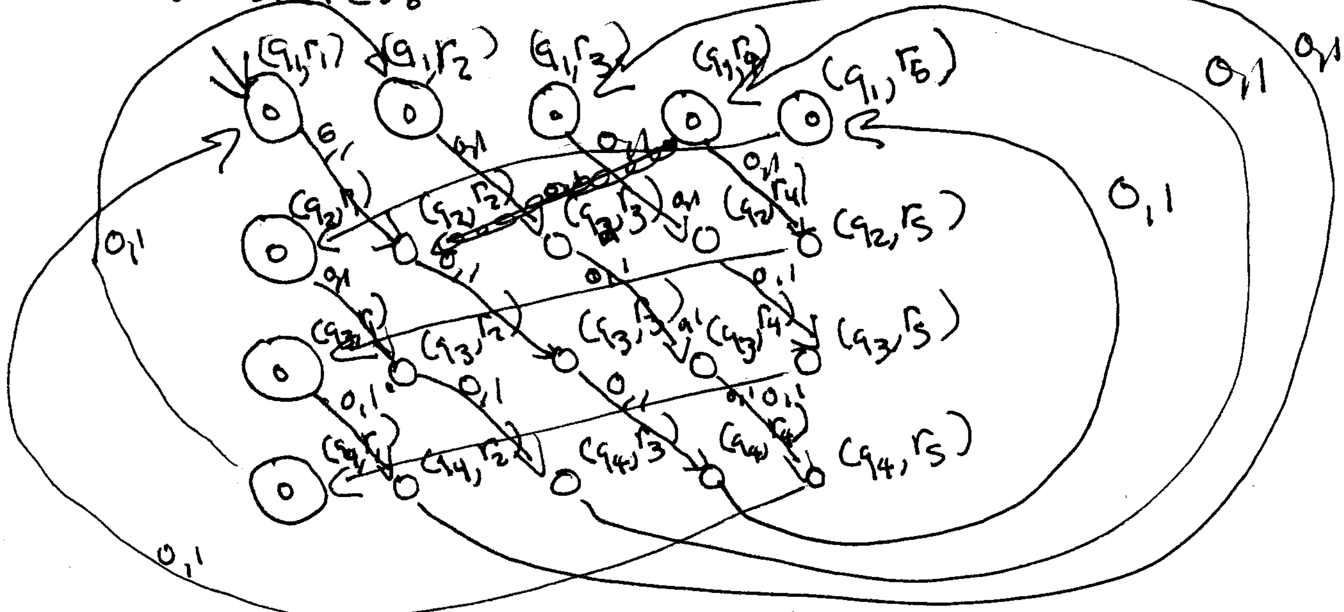
M




M'




Then make machine for $L \cup L'$ using cartesian product of states:



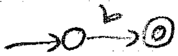

(8) Notice $A^+ = A \circ A^*$

Since $A$ is regular & the regular languages are closed under concatenation and star, $A^+$ will be regular

9) $bb(a \cup b)^* a$

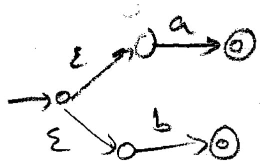
Ramin, Chin, Daniel

$L = \{b\}$

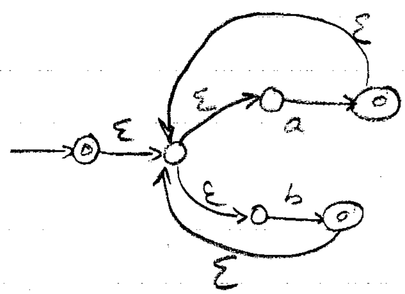



$L = \{b\}$

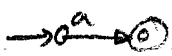

$L = a \cup b$



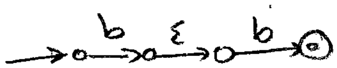

$L = (a \cup b)^*$



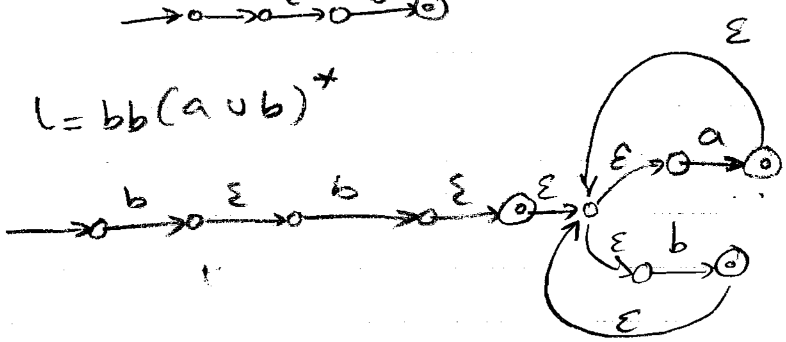

$L = \{a\}$



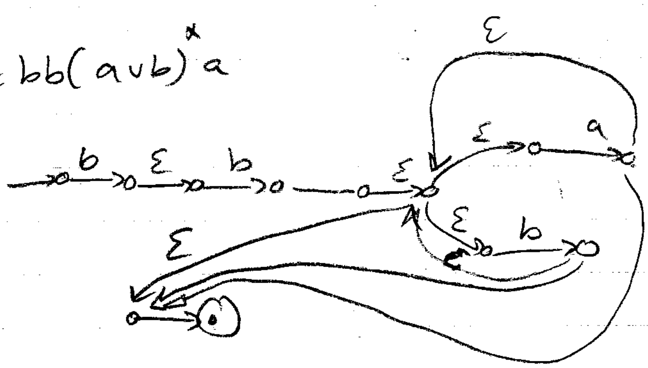

$L = \{bb\}$




$L = bb(a \cup b)^*$




$L = bb(a \cup b)^* a$




#2

on

back side