Turing Machines

CS154
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Outline

- Formal Definition of Turing Machine
- Example

Formal Definition of Turing Machine

- A **Turing Machine** (TM)s a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where
  1. Q is a finite set of states
  2. $\Sigma \subseteq \Gamma$ are respectively the input and tape alphabets. Γ contains a space symbol '_' not in Σ.
  3. δ:Q x Γ --> Q x Γ x {L, R} is the transition function.
  4. $q_0 \in Q$ is the start state.
  5. $q_{accept} \in Q$ is the accept state.
  6. $q_{reject} \in Q$ is the reject state.
- A TM receives its input $w=w_1w_2 \ldots w_n$. on the leftmost n square of its tape. The rest of the tape squares have the blank symbol. The TM's tape head begins on the leftmost square. Once M starts, it follows the rules prescribed by the transition function. A rule

(q,a)-->(q',b, L) says in state q reading an 'a' go to state q', write a 'b' in the current square then move the tape head left. (q,a)-->(q',b, R) would say the same thing except moves the tape head right.

- If the machine ever tries to move off the left side of the tape it, the machine stays in the same place even if the transition says L.
- Computation continues until the machine either enters the accept or reject state at which point the input is either accepted or rejected.

Configurations, Yields

- To specify the state of a computation at a given time (i.e., a **configuration**) one needs to specify the tape contents, the head position, and the current state.
- To do this it suffices to consider only the non-blank square.
- One can use the notation u q v to represent this information. Here u is a string that represents the contents of the tape to the left of the tape head, q is the current state and v is a string consisting of what is under the tape head followed by the non-blank symbols to the right of the tape head.
- For example, one might have $0011q_7 1100$. This says the tape contents are 00111100, the machine is in state q_7, and it is read the third 1 in this string.
- We say configuration C **yields** C' if the TM can legally go from C to C' in one step. For instance, ua q bv yields u q' acv if $\delta(q,b) = (q', c, L)$.

Accept, Reject, Recognize, Decide

- The **start configuration** of M on input w is the configuration q_0 w.
- An **accepting configuration** is one in which the state of the configuration is q_{accept}.
- A **rejecting configuration** is one in which the state of the configuration is q_{reject}.
- Any accepting or rejecting configuration is a **halting configuration** -- one after which the computation halts.
- A Turing machine M **accepts** w if a there is a sequence of configurations C_1, C_2, .., C_k such that C_1 is the start configuration of M on w; for i between 1 and k-1, C_i yields C_{i+1} and C_k is an accept configuration.
- The collection of strings M accepts is denoted L(M) and is called the **language recognized by M**.
- A language is **Turing-recognizable** if some TM recognizes it.
- A language is called **decidable** if there is some TM which halts on all inputs which recognizes it.

Example

- Consider the language $A = \{0^{2^n} | n \geq 0\}$
- Here is a high level description of a machine M recognizing it:

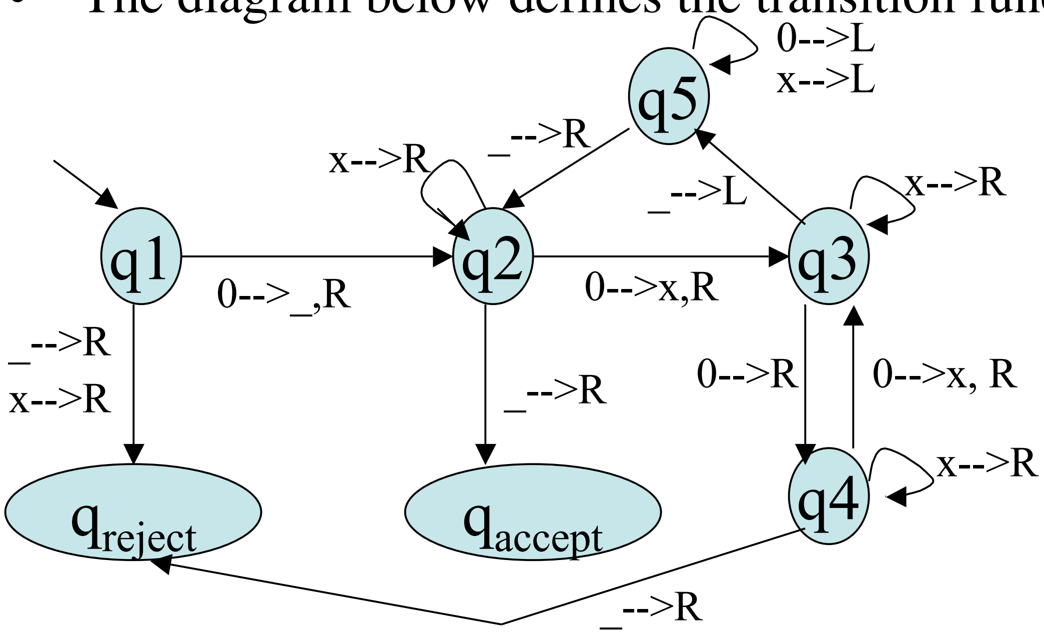
On input w:

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0, accept.
3. If in stage 1 the tape contained more than a single 0 and the number of 0's was odd, reject.
4. Return the head to the left hand side of the tape.
5. Go to stage 1.

- We now try to define M formally.

More on the Example.

- Let Q = {q1,q2, q3, q4, q5}
- Let Σ ={0},
- Let Γ = {0, x, _}
- Let the start, accept, and reject states be q1, q_{accept}, q_{reject}.
- The diagram below defines the transition function:



Here the label on the edge between q1 and q2 corresponds to the transition (q1, 0) --> (q2, _, R). The label on the edge between q3 and q4 corresponds to the transition (q3, _) --> (q5, _, L).

Yet More on the Example

- The machine above on the input 0000 would compute as follows:

$q_1 0000$	$\_q_5 x0x\_$	$\_xq_5 xx\_$
$\_q_2 000$	$q_5 \_x0x\_$	$\_q_5 xxx\_$
$\_xq_3 00$	$\_q_2 x0x\_$	$q_5 \_xxx\_$
$\_x0q_4 0$	$\_xq_2 0x\_$	$\_q_2 xxx\_$
$\_x0xq_3 \_$	$\_xxq_3 x\_$	$\_xq_2 xx\_$
$\_x0q_5 x\_$	$\_xxxq_3 \_$	$\_xxq_2 x\_$
$\_xq_5 0x\_$	$\_xxq_5 x\_$	$\_xxxq_2 \_$
		$\_xxx\_q_{accept}$