Graphs; Strings and Languages; Boolean Logic; Proofs.

CS154

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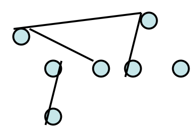
Outline

- Graphs
- Strings and Languages
- Boolean Logic
- Definitions, Theorems, Proofs

Graphs

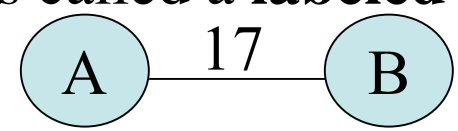
- An **undirected graph** (or just a **graph**) is a set of points with lines connecting some of the points. The points are sometimes called **nodes** or **vertices**.



- Formally, could view as an ordered pair (V, E) where V is a set of vertices and E is a set of sets of the form {v,w} where v and w are in V.
- A **directed graph (digraph)** has edges which are directed from one node to another one. To achieve this using sets, have E be a set of ordered pairs (v, w).

More Graphs

- The **degree** of a vertex is the number of edges that go into it. For directed graphs have both an **in-degree** and an **out-degree**.
- Both edges and vertices in a graph can be **labeled**. In which case the graph is called a **labeled graph**.



- A subgraph of a graph (V, E) is a graph (V', E') such that V' is a subset of V and E' is a subset of E.
- A **path** is a sequence of nodes connected by edges.
- A **simple path** is a path that does not repeat edges.

Still more graphs

- A graph is **connected** if any two points in it are connected by a path.
- In the case of digraphs, one has **directed paths,** and the notion of being **strongly connected**.
- A **cycle** is a path which begins and ends at the same node. A cycle is **simple** if it does not repeat nodes except the end point twice.
- A **tree** is a connected graph without cycles. In such a graph one may designate a **root** node, and any other node with degree 1 is called a **leaf**.

Strings

- Strings of characters are one of the fundamental building blocks of computer science.
- For this class, we will define an **alphabet** to be some nonempty finite set. For example,

  $\Sigma = \{0, 1\}$

  $\Sigma = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$
- The members of this set are called the **symbols** of the alphabet.
- A **string** over an alphabet is a finite sequence of symbols from that alphabet. For example, 0100.
- The **length** of a string w, $|w|$ is the number of symbols it contains.
- The **empty string** is written as ε.

Strings and Languages

- The **reverse** of a string w, w^R, is the string consistent of the symbols of w in reverse order: $001^R = 100$.
- A string is z is a **substring** of w if z appear consecutively within w. So 011 is a substring 10**011**01.
- The **concatenation** of two string x and y, xy, is the string consisting of the symbols in x followed by the symbols of y.
- We write x^k to denote x concatenated to itself k times.
- A **language** is a set of string.

Boolean Logic

- Is a logic based on two values TRUE or FALSE. (sometimes we use 1 and 0).
- These are called **Boolean values**.
- We also have boolean operations:

  NOT (negation) $\neg x$ is TRUE iff x is false

  AND (conjunction) $x \wedge y$ iff both x and y are true

  OR (disjunction) $x \vee y$ iff at least one of x or y is true

  XOR (exclusive OR) $x \oplus y$ iff exactly one of x or y is true

  Equality $x \leftrightarrow y$ iff x and y have the same value

  Implications $x \rightarrow y$ iff x is less than or equal to y

Definitions, Theorems, Proofs

- **Definitions** describes the objects and notions that we use. We want our definitions to be as precise as possible.
- Once we have made some definitions we make **mathematical statements** involving them.
- A **proof** is a convincing logical argument that a statement is true.
- A **theorem** is a mathematical statement which has been proved true.
- A **lemma** is a simple mathematical statement which has been proved true and which will be used in the proof of a theorem.
- A **corollary** is a mathematical statement which can be proved easily once some theorem is known.

Finding proofs

- **Example:** For every graph G, the sum of the degrees of all the nodes in G is an even number.
  - Might approach problem by checking cases like when graph has a small number of vertices. One might then notice each edge contributes two to the total sum and that the sum of degrees = 2* number of edges in graph.
- **Types of proofs**:
  - by construction: example, there is a graph consisting of n nodes with only one cycle. Proof: let V={1,..,n}, let E={{1,2}, {2,3}, …{n-1,n}}∪{{1,n}}.
  - by contradiction: example irrationality of $2^{1/2}$. Idea

    if not can assume $2^{1/2} = m/n$ where m and n share no common factor. In which case, one is odd, the other even. Squaring both sides gives: $2n^2 = m^2$, so m is even because square of an odd number is odd. So m=2k, so $2n^2 = (2k)^2 = 4k^2$. So $n^2 = 2k^2$ implying n is also even, giving a contradiction.
  - by induction:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

You should try to work out.