Blobby Objects and Splines

CS116B

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Outline

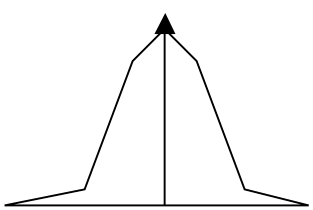
- Blobby Objects
- Spline Representations

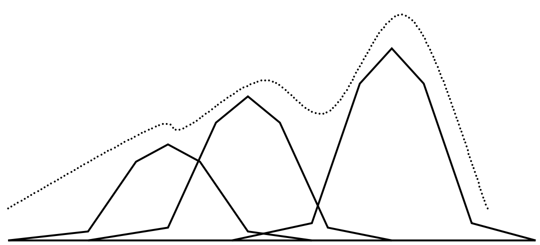
Blobby Objects

- By a blobby object we mean a nonrigid object. That is things, like cloth, rubber, liquids, water droplets, etc.
- These objects tend to exhibit a degree of fluidity.
- For example, in a chemical compound electron density clouds tend to be distorted by the presence of other atoms/molecules.

More Blobby Objects

- Several models have been developed to handle these kind of objects.
- One technique is to use a combination of Gaussian density functions (Gaussian bumps).





- A surface function could then be defined by:

  $f(x,y,z) = \sum_k b_k * \exp(-a_k * r_k^2) - T = 0$

  where $r_k^2 = x_k^2 + y_k^2 + z_k^2$

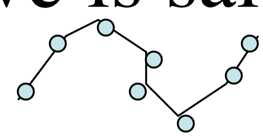
Still More Blobby Objects

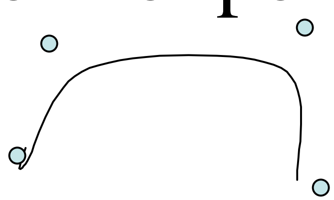
- Another technique called the **meta-ball** technique is to describe the object as being made of density functions much like balls.
- The advantage here is that the density function falls of in a finite interval.

Spline Representations

- In drafting, a **spline** is a flexible strip used to produce a smooth curve through a designated set of points.
- Small weights would be distributed along the strip to hold it in position on the drafting table.
- A **spline curve** and a **spline surface** were originally curves or surfaces created in this manner.
- In computer graphics a spline is any composite curve with polynomial section satisfying some specified continuity conditions
- Over the next day or so we will consider several different kinds of splines.

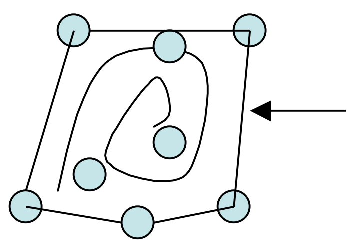
Interpolation and Approximation Splines

- To specify a spline we need to give a set of coordinate positions called **control points.**
- If the polynomial sections are fitted so that all the control points are passed through by the curve then the resulting curve is said to **interpolate** the control points. 
- Otherwise, the curve is said to **approximate** the control points



Convex Hull

- A set of control points defines a **convex hull** in which the spline will live.
- The convex hull of a set of points is the smallest **convex set** containing those points.
- A convex set S is a set of points such that if x, y are in S so is any point on the line between them

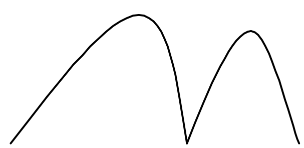


Convex hull

of the control points

Parametric Continuity Conditions

- To ensure a smooth transition from one section of a spline to the next, we can impose various kinds of **continuity conditions**.
- Suppose each section of the spline curve is defined by equations like x=x(u), y=y(u), z(u) for u_1<=u<=u_2
- Then **parametric continuity** would be the condition that at the endpoints of the sections the first derivatives of the two sections match.
- **Zero-order parametric continuity** means the sections simply meet at the endpoints
- **nth-order parametric continuity** means the first n derivatives of the the end points match where sections are connected.



0th order but not 1st order

Geometric Continuity Conditions

- Geometric continuity conditions are a relaxation of parametric continuity conditions.
- Rather than require derivatives to exactly match at end points we only require that the derivatives are proportional to each other.
- Again one can define nth order geometric continuity by requiring the first n derivatives of pairs of endpoints to be proportional to each other.

Spline Specifications

- There are three different ways to specify splines:
  - state the set of boundary conditions that are imposed on the spline
  - state the matrix that characterizes the spline
  - state a set of blending/basis function that determines how specified constraints on the curve are combined to calculate positions along the curve path.

Boundary Condition Example

- Suppose we have the cubic:

  $x(u) = a_x*u^3 + b_x*u^2 + c_x*u + d_x, 0<=u<=1.$
- Boundary conditions might be the coordinates of x(0), x(1) and the derivatives x'(0), x'(1). These four numbers are enough to fix a_x, b_x, c_x, d_x
- Next day will talk about how this representation is connected to the other representations