CS 154
Formal Languages and Computability
Midterm #2 Solutions

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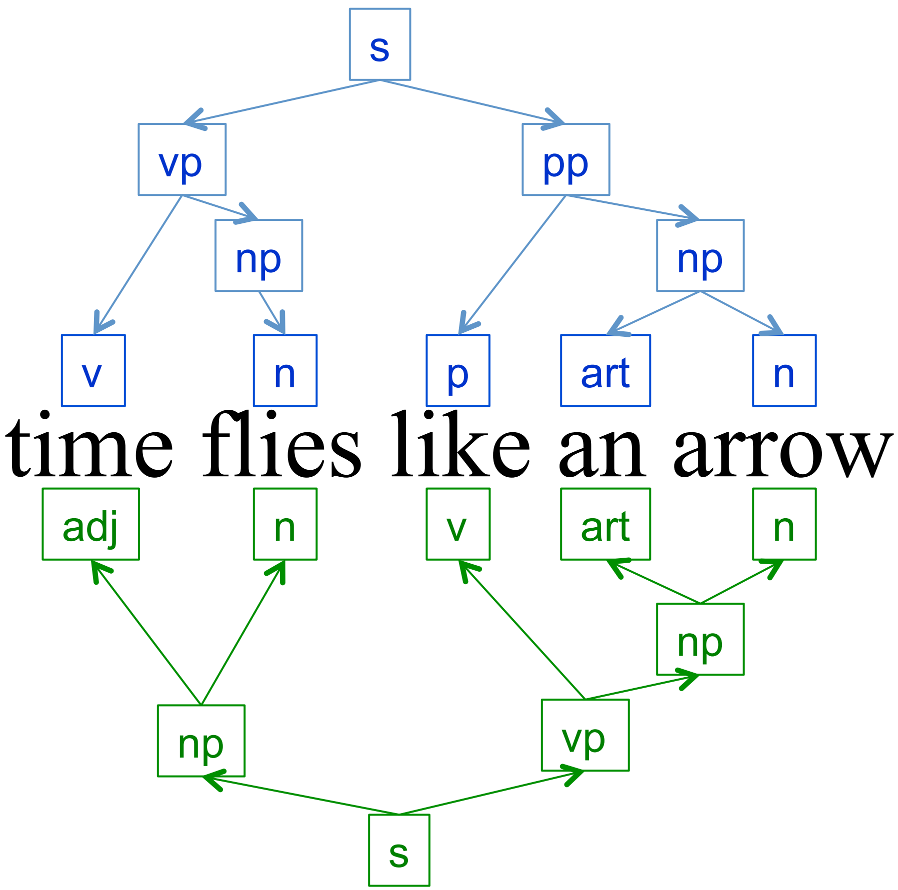
Midterm #2 Solutions: Question 1

- Show two different derivation trees.

<s> → <np><vp> | <vp><pp>
<np> → <adj><n> | <art><n> | <n>
<vp> → <v><np>
<pp> → <p><np>
<adj> → time
<art> → an
<n> → arrow | flies | time
<p> → like
<v> → like | time



Midterm #2 Solutions: Question 2

- There is no way that the PDA's stack can remember an input sequence of symbols in its original order.

Midterm #2 Solutions: Question 3a

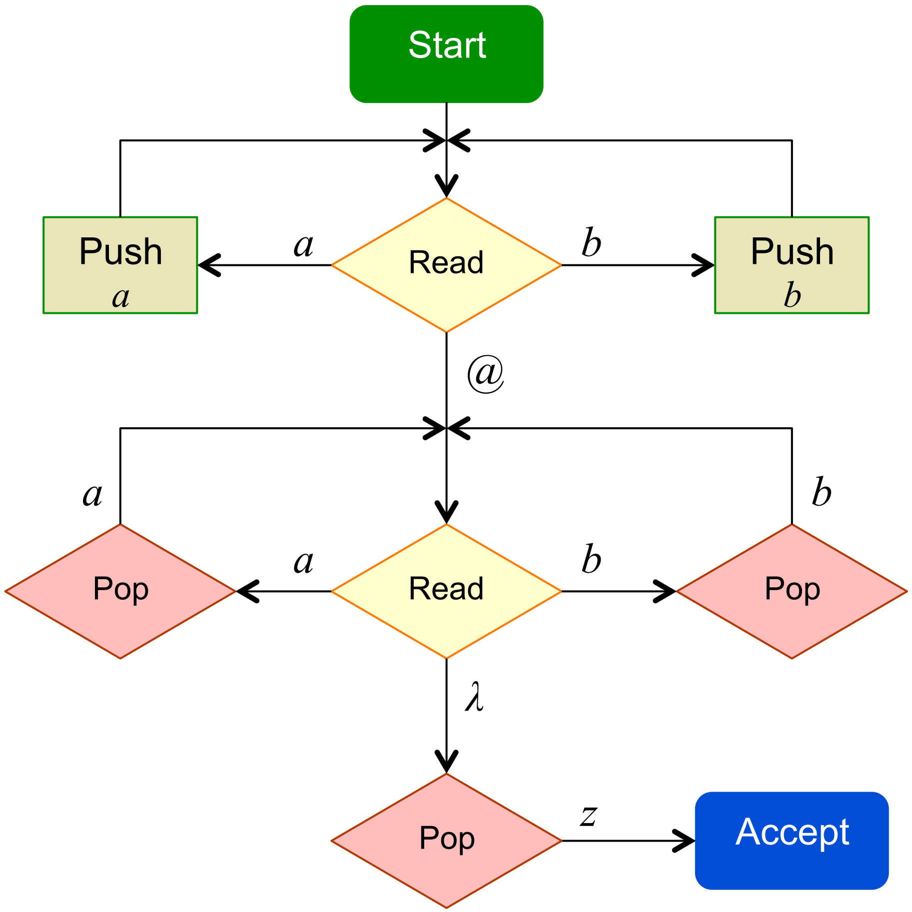
- Language L_1 is nondeterministic because its PDA cannot know where is the middle of an input string (i.e., where w ends and w^R begins), and so the PDA must do backtracking to accept or reject the string.

- L_3 is deterministic because the @ symbol allows its PDA to know exactly where w ends and w^R begins, and so the PDA does not need to backtrack.

Midterm #2 Solutions: Question 3b

$\Gamma = \{a, b, z\}$

stack start symbol = z



Midterm #2 Solutions: Question 4

- Super Calculator *G* = {*V*, *T*, *S*, *P*}
  - *V* = { *<expression>*, *<simple expression>*, *<term>*, *<power>*, *<factor>*, *<relop>*, *<addop>*, *<mulop>*, *<number>* *<digits>* *<E>*, *<sign>* }
  - *T* = { `+ - * / ^ == != < <= > >= && || !` `( ) 0 1 2 3 4 5 6 7 8 9 . E e` }
  - *S* = *<expression>*

Midterm #2 Solutions: Question 4, *cont'd*

- Super Calculator $G = \{V, T, S, P\}$ *cont'd*
  - P is the set of productions

<expression> → *<simple expression>*
| *<simple expression>* *<relop>* *<simple expression>*

<relop> → **==** | **!=** | **<** | **<=** | **>** | **>=**

<simple expression> → *<term>* (*<addop>* *<term>*)*

<addop> → **+** | **-** | **||**

<term> → *<power>* (*<mulop>* *<power>*)*

<mulop> → ***** | **/** | **&&**

<power> → *<factor>* (**^** *<power>*)*

<factor> → *<number>* | **-** *<factor>* | **!** *<factor>* | **(** *<expression>* **)**

<number> → *<digits>* | *<digits>* **.** *<digits>*
| *<digits>* *<E>* *<sign>* *<digits>*
| *<digits>* **.** *<digits>* *<E>* *<sign>* *<digits>*

<digits> → (**0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9**)+

<E> → **e** | **E**

<sign> → **+** | **-** | λ

Midterm #2 Solutions: Question 5

- Let $G = \{V, T, S, P\}$ be the grammar for language L. Since L is context-free, its productions in P all have the form $A \rightarrow x$ where $A \in V$ and $x \in (V \cup T)^*$.

- Construct the context-free grammar $G^T = \{V, T, S, P^T\}$ for language L^T where each production in P_T is $A \rightarrow x^R$. Then for every sentential for w of grammar G, w^R is a sentential form for grammar G^T.

Midterm #2 Solutions: Question 6

- Each string in the language $L = \{a^n b a^{2n} b a^{3n} : n \geq 0\}$ consists of three groups of a's separated by b's, and the lengths of the groups of a's are in the ratio 1 : 2 : 3. Decompose a string as $uvxyz$.

- Neither v nor y can contain a b, otherwise pumping will change the number of b's. Therefore, at least one group of a's is not contained in either v or y. Pumping will break the ratio 1 : 2 : 3. The pumping lemma is violated, and so L cannot be context-free.