Show two different derivation trees.

```
<s>  ⇒ <np><vp> | <vp><pp>
<np> ⇒ <adj><n> | <art><n> | <n>
<vp> ⇒ <v><np>
<pp> ⇒ <p><np>
<adj> ⇒ time
<art> ⇒ an
<n>  ⇒ arrow | flies | time
<p>  ⇒ like
<v>  ⇒ like | time
```

```
time flies like an arrow
```
Midterm #2 Solutions: Question 2

- There is no way that the PDA’s stack can remember an input sequence of symbols in its original order.
Midterm #2 Solutions: Question 3a

- Language $L_1$ is nondeterministic because its PDA cannot know where is the middle of an input string (i.e., where $w$ ends and $w^R$ begins), and so the PDA must do backtracking to accept or reject the string.

- $L_3$ is deterministic because the $@$ symbol allows its PDA to know exactly where $w$ ends and $w^R$ begins, and so the PDA does not need to backtrack.
Midterm #2 Solutions: Question 3b

\[ \Gamma = \{a, b, z\} \]

stack start symbol = \( z \)

---

\[ \text{Start} \]

\[ \text{Read} \]

- Push \( a \)
- Read \( b \)

\[ \text{Pop} \]

- \( a \) to \( b \)

\[ \text{Pop} \]

- \( b \) to \( \lambda \)

\[ \text{Pop} \]

- \( z \) to \( \text{Accept} \)
Midterm #2 Solutions: Question 4

- **Super Calculator** \( G = \{V, T, S, P\} \)
  - \( V = \{<\text{expression}>, <\text{simple expression}>, <\text{term}>, <\text{power}>, <\text{factor}>, <\text{relop}>, <\text{addop}>, <\text{mulop}>, <\text{number}>, <\text{digits}>, <E>, <\text{sign}>\} \)
  - \( T = \{+,-,\ast,\div,\wedge,\text{==,!=,<,<=,>,>=,&&,||,!} (, ) 0 1 2 3 4 5 6 7 8 9 . E e\} \)
  - \( S = <\text{expression}> \)
Super Calculator $G = \{V, T, S, P\}$ cont’d

$P$ is the set of productions

\[
\begin{align*}
<\text{expression}> & \rightarrow <\text{simple expression}> \\
& \quad | <\text{simple expression}> <\text{relop}> <\text{simple expression}>
\end{align*}
\]

\[
<\text{relop}> \rightarrow == | != | < | <= | > | >=
\]

\[
<\text{simple expression}> \rightarrow <\text{term}> ( <\text{addop}> <\text{term}> )* \\
<\text{addop}> \rightarrow + | - | || \\
<\text{term}> \rightarrow <\text{power}> ( <\text{mulop}> <\text{power}> )* \\
<\text{mulop}> \rightarrow * | / | && \\
<\text{power}> \rightarrow <\text{factor}> ( ^ <\text{power}> )* \\
<\text{factor}> \rightarrow <\text{number}> | - <\text{factor}> | ! <\text{factor}> | ( <\text{expression}> ) \\
<\text{number}> \rightarrow <\text{digits}> | <\text{digits}> . <\text{digits}> \\
& \quad | <\text{digits}> <E> <\text{sign}> <\text{digits}> \\
& \quad | <\text{digits}> . <\text{digits}> <E> <\text{sign}> <\text{digits}>
\]

\[
<\text{digits}> \rightarrow ( 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 )+
\]

\[
<\text{E}> \rightarrow e | E \\
<\text{sign}> \rightarrow + | - | \lambda
\]
Midterm #2 Solutions: Question 5

- Let $G = \{V, T, S, P\}$ be the grammar for language $L$. Since $L$ is context-free, its productions in $P$ all have the form $A \rightarrow x$ where $A \in V$ and $x \in (V \cup T)^*$. 

- Construct the context-free grammar $G^T = \{V, T, S, P^T\}$ for language $L^T$ where each production in $P_T$ is $A \rightarrow x^R$. Then for every sentential for $w$ of grammar $G$, $w^R$ is a sentential form for grammar $G^T$. 
Midterm #2 Solutions: Question 6

- Each string in the language \( L = \{a^n b a^{2n} b a^{3n} : n \geq 0\} \) consists of three groups of \( a \)'s separated by \( b \)'s, and the lengths of the groups of \( a \)'s are in the ratio 1 : 2 : 3. Decompose a string as \( uvxyz \).

- Neither \( v \) nor \( y \) can contain a \( b \), otherwise pumping will change the number of \( b \)'s. Therefore, at least one group of \( a \)'s is not contained in either \( v \) or \( y \). Pumping will break the ratio 1 : 2 : 3. The pumping lemma is violated, and so \( L \) cannot be context-free.