Recursively enumerable and recursive languages

1. [10 points] Use Turing machines to show that the set of \textit{recursively enumerable} languages is closed under union and intersection.

2. [10 points] Use Turing machines to show that the set of \textit{recursive} languages is closed under union and intersection.

3. [10 points] Show that the set of \textit{recursive} languages is closed under reversal. Closed under reversal means that if a language \( L \) is recursive, then the language \( L^R \) containing all the strings of \( L \) reversed is also recursive.

4. [10 points] Suppose language \( L \) is accepted by a \textit{nondeterministic} Turing machine that always halts on any input string. Show that \( L \) is recursive.

5. [20 points] Suppose a language \( L \) has a function \( f \) such that \( f(w) = 1 \) if \( w \in L \) and \( f(w) = 0 \) otherwise. Show that function \( f \) is Turing-computable if and only if the language \( L \) is recursive.
6. [20 points] Let $D$ be a recursive language of string pairs $<x, y>$. Let $C$ be the set of all strings $x$ for which there exists some $y$ such that $<x, y> \in D$. Show that $C$ is recursively enumerable.

7. [20 points] Let $C$ be a recursively enumerable language. Show that there exists a recursive language $D$ of string pairs (see Problem 6) such that $C$ contains exactly the strings $x$ such that there exists some $y$ such that $<x, y> \in D$.

What to submit to Canvas

Submit your answers in a Word document or PDF into Canvas: Assignment #7