Problem set B

1. Use the pumping lemma to show that the language
   \( L = \{a^n : n \text{ is a perfect cube: } 0, 1, 8, 27, \ldots \} \) is not regular.

2. Use the pumping lemma to show that the language
   \( L = \{a^n : n \text{ is a power of } 2: 1, 2, 4, 8, \ldots \} \) is not regular.

3. Use the pumping lemma to show that the language
   \( L = \{a^p : p \text{ and } q \text{ are both prime numbers} \} \) is not regular.

4. Use the pumping lemma to show that the language
   \( L = \{a^p a^q : p \text{ divided by } q \text{ is an integer quotient} \} \) is not regular.

5. Use the pumping lemma to show that the language
   \( L = \{a^p a^q : p + q \text{ is a prime number} \} \) is not regular.

6. Let \( \Sigma = \{0, 1, +, =\} \). Use the pumping lemma to show that the language
   \( L = \{x=y+z : x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z \} \) is not regular.
   For example, the string 1001=10+111 is in \( L \).

7. Let language \( L \) be denoted by the regular expression \( a^* b^* \)
   What is wrong with the following “proof” that \( L \) is not regular?

Assume that \( L \) is regular. Then it must be defined by a DFA with \( k \) states, for some integer \( k > 0 \). Take the string \( w = a^k b^k \) and split it \( w = xyz \), with \( y = ab \). Then \( wy^2z \) is not in \( L \), which contradicts the pumping lemma. Therefore, \( L \) cannot be regular.
8. Prove whether or not language \( L = \{ a^{p+qi} : p \text{ and } q \text{ are fixed integer values, and } i \geq 0 \} \) is regular.

9. Prove whether or not language 
\( L = \{ a^p b^q : p \geq 100 \text{ and } q \geq 100 \text{ are fixed integer values, and } i \geq 0 \} \) is regular.

10. Assume that \(<\text{stmt}>\), \(<\text{if_stmt}>\), \(<\text{boolexpr}>\), and \(<\text{assign_stmt}>\) are nonterminal symbols, and if, else, (, and ) are terminal symbols.

Here’s a grammar written in BNF for Java-style IF statements.

\[
<\text{stmt}> ::= <\text{assign_stmt}> \mid <\text{if_stmt}>
\]

\[
<\text{if_stmt}> ::= \text{if} ( <\text{boolexpr}> ) <\text{stmt}>
\mid \text{if} ( <\text{boolexpr}> ) <\text{stmt}> \text{ else } <\text{stmt}>
\]

How is this grammar ambiguous? Give an example of an ambiguity.

**What to submit to Canvas**

Submit your answers as a Word document or a PDF into Canvas: **Assignment #3**