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On Edge-Balance Index Sets of Flower Graphs

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Abstract

Let $G = (V, E)$ be a simple graph, and let $A = \{0, 1\}$. Any edge labeling $f : E \rightarrow A$ induces a partial vertex labeling $f^* : V \rightarrow A$ that assigns 0 or 1 to $f^*(v)$, depending on whether there are more 0- or 1-edges incident to v , and leaves $f^*(v)$ unlabeled otherwise. For each $i \in A$, let $e_f(i)$ and $v_f(i)$ denote the number of edges and vertices, respectively, that are labeled i . The edge-balance index set of G is defined as $\{|v_f(0) - v_f(1)| : |e_f(0) - e_f(1)| \leq 1\}$. In this paper, we obtain the edge-balance index sets of flower graphs, all of them form arithmetic progressions.

1 Introduction

In [1, 7], a new problem in graph labeling was proposed. It can be viewed as the dual of balance index sets, a research topic that has produced many interesting results recently [2]-[6], [8]-[14].

Let $G = (V, E)$ be a simple graph with vertex set V and edge set E . Given any edge labeling $f : E \rightarrow A$, where $A = \{0, 1\}$, we define an associated partial vertex labeling $f^* : V \rightarrow A$ as follows. Define $f^*(v)$ to be 0 if it is incident to more 0-edges than 1-edges, and 1 if it is incident to more 1-edges than 0-edges. If the vertex v is incident to an equal number of 0- and 1-edges, leave it unlabeled. Hence f^* is a partial function. For each $i \in A$, let $e_f(i) = |\{uv \in E : f(uv) = i\}|$, and $v_f(i) = |\{v \in V : f^*(v) = i\}|$. If there is no ambiguity we would drop the subscript and simply write $e(i)$ and $v(i)$ respectively.

A graph G is said to be **edge-friendly** if it admits an edge labeling f such that $|e_f(0) - e_f(1)| \leq 1$. If, in addition, we also have $|v_f(0) - v_f(1)| \leq 1$, we call G **edge-balanced**. An edge-friendly graph could be far from being edge-balanced. To extend the study of edge-balancedness, we introduce the

notion of an *edge-balance index set*:

$$\text{EBI}(G) = \{ |v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly} \}.$$

In other words, $\text{EBI}(G)$ is the set of values that $|v(0) - v(1)|$ could attain as we go over all edge-friendly labelings of G .

Example 1 The graph nK_2 is the union of n disjoint edges. We claim that

$$\text{EBI}(nK_2) = \begin{cases} \{0\} & \text{if } n \text{ is even,} \\ \{2\} & \text{if } n \text{ is odd.} \end{cases}$$

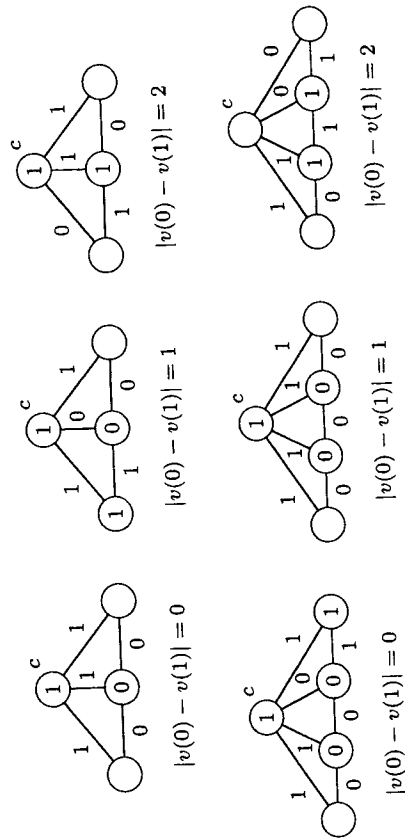
Here is the reason. If an edge is labeled i , then both its end vertices are labeled i as well. If n is even, there is an equal number of 0- and 1-edges, hence an equal number of 0- and 1-vertices. Likewise, if n is odd, the numbers of 0- and 1-edges differ by 1, hence the numbers of 0- and 1-vertices differ by 2. \square

Example 2 The star $\text{St}(n)$ is the tree with diameter two, and n pendant edges incident to the center c . Each of the n pendant vertices is labeled the same way as the edge incident to it. The center is either unlabeled, labeled 0 or labeled 1, depending on whether $e(0) - e(1)$ equals 0, 1, or -1 , respectively. Therefore

$$\text{EBI}(\text{St}(n)) = \begin{cases} \{0\} & \text{if } n \text{ is even,} \\ \{2\} & \text{if } n \text{ is odd,} \end{cases}$$

as one can easily verify. \square

Example 3 Chopra, Lee and Su [1] investigated the edge-balance index sets of fans and wheels. The fan graph $F_{1,n}$ connects a vertex c to every vertex on a path of order n . The fact that $\text{EBI}(F_{1,3}) = \text{EBI}(F_{1,4}) = \{0, 1, 2\}$ is illustrated (using edge-friendly labelings) below. \square

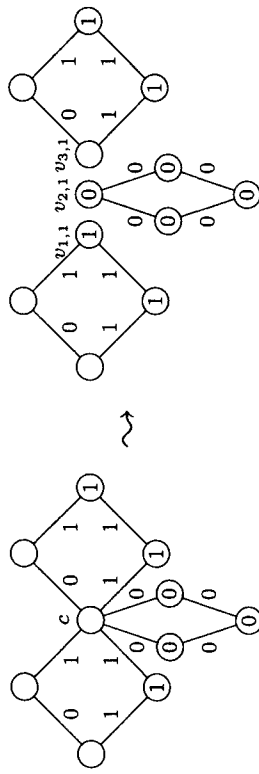


Let (H, x) denote a graph H with a specific vertex x . We construct the graph $\text{Amal}((H, x), n)$, the amalgamation of n copies of H , by identifying all n copies of x . The resulting graph is called the *one-point union of* (H, x) . In particular, for a cycle C_m with vertex set $\{v_1, v_2, \dots, v_m\}$, we will call $\text{Amal}((C_m, v_1), n)$ a *flower graph*. For simplicity we will denote it by $F(m, n)$. Naturally, we can extend it to the one-point union of n cycles of length m_1, m_2, \dots, m_n , which we call the *generalized flower graph* $\text{GF}(m_1, m_2, \dots, m_n)$. In this paper, we investigate the edge-balance index sets of generalized flower graphs.

2 The Preliminary Results

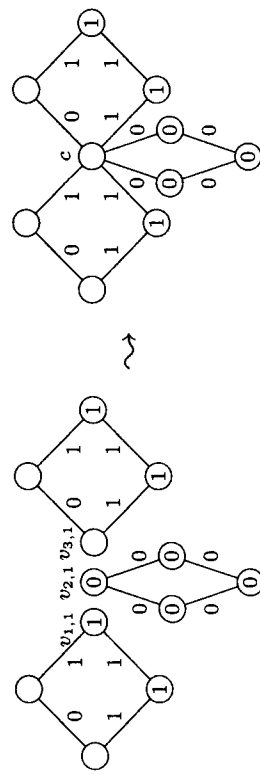
Let the vertices of the i th cycle of $\text{GF}(m_1, m_2, \dots, m_n)$ be $v_{i,1}, v_{i,2}, \dots, v_{i,m_i}$, and assume that $v_{i,1}$ is the specific vertex used in the amalgamation, and we call it the center c of the new graph.

For any edge-labeling f of $F(m_1, m_2, \dots, m_n)$, let f_i denote its restriction on the i th cycle. Then $f_i^*(v_{i,j}) = f^*(v_{i,j})$ if $j \neq 1$.



However $f_i^*(v_{i,1})$ may be different from $f^*(c)$, because it only depends on the labels of the two edges incident to $v_{i,1}$, but $f^*(c)$ depends on the labels of the $2n$ edges incident to c .

Conversely, an edge labeling of $F(m_1, m_2, \dots, m_n)$ can be obtained by "splicing" individual edge-labelings f_i 's (each of which may not be edge-friendly) of the n cycles at the $v_{i,1}$'s, and then adjusting the value of $f^*(c)$ accordingly.



To study the edge-balance index set of $\text{GF}(m_1, m_2, \dots, m_n)$, we first need to investigate the value of $v(0) - v(1)$ that a cycle can assume.

Lemma 2.1 For any edge labeling (which may not be edge-friendly) of a cycle, we have $v(0) - v(1) = e(0) - e(1)$.

Proof. The result is immediate if all edges are given the same label. If the labels consist of both 0s and 1s, we may assume the labeling starts with c_1 0-edges, followed by d_1 1-edges, then c_2 0-edges, d_2 1-edges, and so forth, until it ends with c_b 0-edges and d_b 1-edges. It is easy to verify that

$$v(0) = \sum_{i=1}^b (c_i - 1) = \left(\sum_{i=1}^b c_i \right) - b = e(0) - b,$$

and similarly, $v(1) = e(1) - b$. Thus $v(0) - v(1) = e(0) - e(1)$. \square

If the edge-labeling is edge-friendly, we obtain the following result.

Corollary 2.2 ([7]) For $m \geq 3$,

$$EBI(C_m) = \begin{cases} \{0\} & \text{if } m \text{ is even,} \\ \{1\} & \text{if } m \text{ is odd.} \end{cases}$$

It follows from Lemma 2.1 that, for $\text{GF}(m_1, m_2, \dots, m_n)$,

$$v(0) - v(1) = e(0) - e(1) + \delta,$$

where δ represents the adjustment resulted from the splicing of the n edge labelings f_1, f_2, \dots, f_n .

Assume n_0 of these f_i 's produce $f_i^*(v_{i1}) = 0$, and n_1 of them yield $f_i^*(v_{i1}) = 1$, and n_2 leave $f_i^*(v_{i1})$ unlabeled. Of course, we need $0 \leq n_0, n_1, n_2 \leq n$, and $n_0 + n_1 + n_2 = n$. It is also clear that

$$f^*(c) = \begin{cases} 0 & \text{if } n_0 > n_1, \\ 1 & \text{if } n_0 < n_1, \\ \text{unlabeled} & \text{if } n_0 = n_1. \end{cases}$$

To compute δ , we first discard the contribution of $f_i^*(v_{i1})$ to $v(0) - v(1)$. Since there are n_0 0-vertices and n_1 1-vertices among these $f_i^*(v_{i1})$'s, the removal requires the value of $v(0) - v(1)$ to be adjusted by $n_1 - n_0$. Next, we consider the contribution from $f^*(c)$. We obtain

$$\delta = \delta(n_0, n_1) = \begin{cases} n_1 - n_0 + 1 & \text{if } n_0 > n_1, \\ n_1 - n_0 - 1 & \text{if } n_0 < n_1, \\ 0 & \text{if } n_0 = n_1. \end{cases}$$

After going over all possible values of n_0, n_1 and n_2 , we obtain the possible values of δ , adding which to $e(0) - e(1)$ yields the possible values of $v(0) - v(1)$. For a friendly labeling, we know $e(0) - e(1)$ equals 0 or ± 1 . Therefore, if $N = \sum_{i=1}^n m_i$,

$$v(0) - v(1) = \begin{cases} \delta(n_0, n_1) & \text{if } N \text{ is even,} \\ \delta(n_0, n_1) \pm 1 & \text{if } N \text{ is odd.} \end{cases} \quad (1)$$

It remains to determine the values that $\delta(n_0, n_1)$ can assume.

Since we can pick n_2 such that $n_0 = n_1$, we find that δ could be 0. If $n_0 \neq n_1$, we notice that $n_1 - n_0 - 1 = -(n_0 - n_1 + 1)$. Hence, due to symmetry,

$$\begin{aligned} \{n_1 - n_0 - 1 \mid n_0 + n_1 = n - n_2, n_0 < n_1\} \\ = \{- (n_1 - n_0 + 1) \mid n_0 + n_1 = n - n_2, n_1 < n_0\}. \end{aligned}$$

In other words, the two cases $n_0 > n_1$ and $n_0 < n_1$ produce the same list of values in $|\delta(n_0, n_1)|$ when mm is even, and the same list of values in $|\delta(n_0, n_1) \pm 1|$ when mm is odd. Therefore it suffices to consider only $n_0 < n_1$, which is what we will assume in the rest of our discussion.

From $n_0 = n - n_1 - n_2$ and $\delta(n_0, n_1) = n_1 - n_0 - 1$, we obtain

$$\delta(n_0, n_1) = -(n - n_2) - 1 + 2n_1.$$

It also follows from $0 \leq n_0 = n - n_1 - n_2 < n_1$ that

$$\lfloor (n - n_2)/2 \rfloor \leq n_1 \leq n - n_2$$

if all combinations of n_0, n_1, n_2 are possible, in which case,

$$\delta(n_0, n_1) = \begin{cases} -1, 1, 3, \dots, n - n_2 - 1 & \text{if } n - n_2 \text{ is even,} \\ 0, 2, 4, \dots, n - n_2 - 1 & \text{if } n - n_2 \text{ is odd.} \end{cases}$$

3 Generalized Flower Graphs

The *generalized flower graph* $\text{GF}(m_1, m_2, \dots, m_n)$ is the one-point union of n cycles of length m_1, m_2, \dots, m_n . It has $N = m_1 + m_2 + \dots + m_n$ edges. Recall that we may assume $n_0 < n_1$.

Theorem 3.1 Given $m_1, m_2, \dots, m_n \geq 3$, where $n \geq 2$, if $N = \sum_{i=1}^n m_i$ is even, then

$$EBI(\text{GF}(m_1, m_2, \dots, m_n)) = \begin{cases} \{0, 1, 2, \dots, n - 1\} & \text{if } N \geq 4n, \\ \{0, 1, 2, \dots, N/2 - n - 1\} & \text{if } N \leq 4n - 2. \end{cases}$$

Proof. Since N is even, $v(0) - v(1) = \delta(n_0, n_1)$, where

$$\delta(n_0, n_1) = n_1 - n_0 - 1 = -(n - n_2) - 1 + 2n_1.$$

Recall that $\lfloor (n - n_2)/2 \rfloor \leq n_1$, thus

$$\delta(n_0, n_1) = \begin{cases} -1, 1, 3, \dots & \text{if } n - n_2 \text{ is even,} \\ 0, 2, 4, \dots & \text{if } n - n_2 \text{ is odd.} \end{cases}$$

Once we determine the upper bound for n_1 , we would be able to find the upper bound for $\delta(n_0, n_1)$.

Since we assume $n_0 < n_1$, there are $2(n_1 - n_0)$ more 1-edges than 0-edges that are incident to c . If we can label that many edges among the remaining $N - 2n$ edges with 0, we can easily extend this partially completed labeling to an edge-friendly labeling. This requires $2(n_1 - n_0) \leq N - 2n$. Since $n_0 = n - n_1 - n_2$, we obtain $n_1 \leq \lfloor (N - 2n_2)/4 \rfloor$. Together with the obvious bound $n_1 \leq n - n_2$, we determine that

$$n_1 \leq \min(n - n_2, \lfloor (N - 2n_2)/4 \rfloor).$$

Notice that $n - n_2 \leq \lfloor (N - 2n_2)/4 \rfloor$ if and only if $n - n_2 \leq (N - 2n_2)/4$, which is equivalent to $4n \leq N + 2n_2$. Hence

$$n_1 \leq \begin{cases} n - n_2 & \text{if } 4n \leq N + 2n_2, \\ \lfloor (N - 2n_2)/4 \rfloor & \text{if } 4n > N + 2n_2. \end{cases}$$

It follows immediately that if $N \geq 4n$, we have $n_1 \leq n - n_2$, and

$$\delta(n_0, n_1) = \begin{cases} -1, 1, 3, \dots, n - n_2 - 1 & \text{if } n - n_2 \text{ is even,} \\ 0, 2, 4, \dots, n - n_2 - 1 & \text{if } n - n_2 \text{ is odd.} \end{cases}$$

Setting $n_2 = 0, 1$, we find that

$$\delta(n_0, n_1) = -1, 0, 1, 2, \dots, n - 1 \quad \text{if } N \geq 4n.$$

Next, assume $N \leq 4n - 4$. Then $4n \leq N + 2n_2$ would require $n_2 \geq 2$, and we have just shown that in such an event, $\delta(n_0, n_1) \leq n - n_2 - 1$. Since $4n \leq N + 2n_2$, we also find that $n - n_2 - 1 \leq N/2 - n - 1$. So we have $\delta(n_0, n_1) \leq N/2 - n - 1$.

If $n_2 \leq 1$, we have $n_1 \leq \lfloor (N - 2n_2)/4 \rfloor$. For $N \equiv 0 \pmod{4}$, since $\lfloor (N - 2n_2)/4 \rfloor = N/4 + \lfloor -n_2/2 \rfloor$, we find (for brevity, let $\delta = \delta(n_0, n_1)$)

$$\delta = \begin{cases} -1, 1, 3, \dots, N/2 - n - 1 + n_2 + 2\lfloor -n_2/2 \rfloor & \text{if } n - n_2 \text{ is even,} \\ 0, 2, 4, \dots, N/2 - n - 1 + n_2 + 2\lfloor -n_2/2 \rfloor & \text{if } n - n_2 \text{ is odd.} \end{cases}$$

For $N \equiv 2 \pmod{4}$, we find $\lfloor (N - 2n_2)/4 \rfloor = (N - 2)/4 + \lfloor (1 - n_2)/2 \rfloor$. Hence

$$\delta = \begin{cases} -1, 1, 3, \dots, N/2 - n - 2 + n_2 + 2\lfloor (1 - n_2)/2 \rfloor & \text{if } n - n_2 \text{ is even,} \\ 0, 2, 4, \dots, N/2 - n - 2 + n_2 + 2\lfloor (1 - n_2)/2 \rfloor & \text{if } n - n_2 \text{ is odd.} \end{cases}$$

In both cases, regardless of the parity of n , by setting $n_2 = 0, 1$, we find,

$$\delta(n_0, n_1) = -1, 0, 1, 2, \dots, N/2 - n - 1 \quad \text{if } N \leq 4n - 4.$$

The last unsettled case is $N = 4n - 2$. For $4n \leq N + 2n_2$, we need $n_2 \geq 1$. In particular, when $n_2 = 1$, we find $n_1 \leq n - 1$, therefore $\delta(n_0, n_1) \leq n - 2 = N/2 - n - 1$. For $4n > N + 2n_2$, we need $n_2 = 0$. From what we have just derived above, we conclude that $\delta(n_0, n_1) \leq N/2 - n - 2$. Therefore

$$\delta(n_0, n_1) = -1, 0, 1, 2, \dots, N/2 - n - 1 \quad \text{if } N = 4n - 2.$$

Combining these observations yields the desired result. \square

Example 4 To illustrate Theorem 3.1, it suffices to show an edge-labeling for each value within the EBI set. To save space, instead of displaying the graphs, we only describe the edge labels of each cycle, starting with an edge that is incident to the center c . \square

graph	edge labels	$v(0) - v(1)$
GF(3, 4, 5)	000 0001 11111	0
	000 0110 01111	1
	010 0110 01110	2
GF(3, 3, 4, 4)	000 000 1011 1111	0
	000 101 1001 1011	1
	001 101 1001 1001	2
GF(3, 3, 4, 8)	000 000 0001 11111111	0
	000 000 0010 11111111	1
	000 000 0110 01111111	2
	000 010 0110 01111110	3

Theorem 3.2 Given $m_1, m_2, \dots, m_n \geq 3$, where $n \geq 2$, if $N = \sum_{i=1}^n m_i$ is odd, then

$$EBI(GF(m_1, \dots, m_n)) = \begin{cases} \{0, 1, 2, \dots, n\} & \text{if } N \geq 4n + 1, \\ \{0, 1, 2, \dots, (N - 1)/2 - n\} & \text{if } N \leq 4n - 1. \end{cases}$$

Proof. When N is odd, we have

$$v(0) - v(1) = e(0) - e(1) + \delta(n_0, n_1) = \delta(n_0, n_1) \pm 1.$$

To maximize $v(0) - v(1)$, we may assume $e(0) - e(1) = 1$. Incident to the center c , there are $2(n_1 - n_0)$ more 1-edges than 0-edges. If we can label $2(n_1 - n_0) + 1$ edges among the remaining $N - 2n$ edges with 0, we can easily extend this partially completed labeling to an edge-friendly labeling. Therefore we need $2(n_1 - n_0) + 1 \leq N - 2n$.

The determination of $\delta(n_0, n_1)$ proceeds in the same manner as we did in the case of even N , except that all occurrences of N need to be replaced by $N-1$. We find

$$\delta(n_0, n_1) = \begin{cases} -1, 1, 3, \dots, n-1 & \text{if } N \geq 4n+1, \\ 0, 2, 4, \dots, (N-1)/2 - n - 1 & \text{if } N \leq 4n-1. \end{cases}$$

Since $v(0) - v(1) = \delta(n_0, n_1) + 1$, we obtain the stated result. \square

Example 5

graph	edge labels	$ v(0) - v(1) $
GF(3, 3, 5)	000 010 0111	0
	000 000 1111	1
	001 101 10001	2
GF(3, 3, 4, 5)	000 000 0110 1111	0
	000 000 0011 1111	1
	000 101 1001 10011	2
	001 101 1001 10001	3
GF(3, 3, 3, 8)	000 000 000 1111111	0
	000 000 001 1011111	1
	000 010 010 0111110	2
	001 101 101 1000011	3
	101 101 101 10000001	4

Corollary 3.3 For $n \geq 2$,

$$EBI(F(3, n)) = \begin{cases} \{0, 1, 2, \dots, k-1\} & \text{if } n = 2k, \\ \{0, 1, 2, \dots, k\} & \text{if } n = 2k+1, \end{cases}$$

and if $m \geq 4$,

$$EBI(F(m, n)) = \begin{cases} \{0, 1, 2, \dots, n-1\} & \text{if } mn \text{ is even,} \\ \{0, 1, 2, \dots, n\} & \text{if } mn \text{ is odd.} \end{cases}$$

Proof. Since $n \geq 2$, we have $3n \leq 4n - 2$. Therefore, when $N = 3n$, the upper bound for $|v(0) - v(1)|$ is

$$\begin{cases} N/2 - n - 1 = 3k - 2k - 1 = k - 1, & \text{if } n = 2k, \\ (N-1)/2 - n = 3k + 1 - (2k + 1) = k. & \text{if } n = 2k + 1. \end{cases}$$

For $m \geq 4$, we find

$$N = mn \geq \begin{cases} 4n & \text{if } mn \text{ is even,} \\ 4n + 1 & \text{if } mn \text{ is odd.} \end{cases}$$

The result follows immediately from Theorems 3.1 and 3.2. \square

Example 6

graph	edge labels	$ v(0) - v(1) $
F(3, 2)	010 011	0
	010 010 011	0
F(3, 3)	000 010 111	1
	000 010 011 111	0
F(3, 4)	010 010 010 111	1
	000 010 010 011 111	0
F(3, 5)	000 000 010 111 111	1
	000 001 101 101 101	2

Example 7

graph	edge labels	$ v(0) - v(1) $
F(4, 2)	0000 1111	0
	0110 0110	1
F(4, 3)	0000 0011 1111	0
	0001 1001 1011	1
	0110 0110 0110	2
F(5, 3)	00000 01110 01111	0
	00000 00011 11111	1
	00001 10001 10111	2
	10001 10001 10011	3
F(5, 4)	00000 00000 11111 11111	0
	00000 00110 01110 11111	1
	00000 01110 10001 01111	2
	00100 01110 01110 01110	3
F(5, 5)	00000 00000 01110 01111 11111	0
	00000 00000 00011 11111 11111	1
	00000 00001 10001 10111 11111	2
	00000 10001 10001 10011 11111	3
	00001 10001 10001 10001 11111	4
10001 10001 10001 10001 10111	5	

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**Forty-Second Southeastern International Conference
on Combinatorics, Graph Theory, & Computing***
Florida Atlantic University
March 7-11, 2011

I have just returned from the latest in the series of Southeastern conferences, held once again in sunny Boca Raton, Florida. The conference was well-attended, and a success both for the level of organization and the quality of papers presented.

As in the past, conference registration in the foyer to the Grand Palm Room was very efficient, and refreshments were served inside. Later, and throughout the week, publishers from three companies manned booths in the coffee area.

Monday morning, the conference Opening Session began with greetings and remarks from FAU President Saunders, Provost Alpering, and Dean Perry.

Plenary talks were given Monday to Thursday in the Grand Palm Room of the Student Union, and were very well attended. The invited speakers were: Sue Whitesides (University of Victoria); Martin Golumbic (University of Haifa); Stephen Locke (Florida Atlantic University); and Uwe Leck (University of Wisconsin-Superior)

The conference roster listed over 240 participants, with 140 contributed talks covering a wide range of topics scheduled from Monday morning to Friday noon. Contributed talks were presented in the Live Oak Pavillion located adjacent to the Student Union building. I had the pleasure of chairing one of the sessions, and was very impressed by the content of the talks, and that each speaker kept to the time allowed for their presentation and questions.

Wednesday morning the traditional conference group photo was taken, and after the coffee break the Annual General Meeting for the Institute of Combinatorics and Its Applications was held in the Grand Palm Room. Following the AGM, a memorial talk in honor of Ralph Stanton was presented by Ron Mullin, Wendy Myrvold, and Ernie Ruet d'Auteuil.

On Wednesday afternoon, the plenary talk was replaced by a Session on Unsolved Problems. Quite a few challenging ideas were entertained, which we hope will generate interesting results in the future.

Following the time-tested pattern, the conference organizers treated participants to coffee and snacks every day, and several evening gatherings. On Monday there was a late afternoon social event held at the beautiful new "green" Faculty Club on campus. Tuesday was the outdoor social party at the Live Oak Pavillion

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