Pairwise Alignment

- Given two strings S and T over some alphabet.

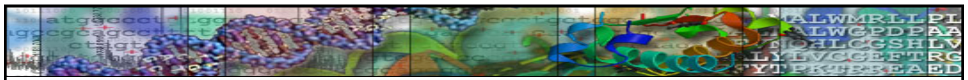
  Length of S is $|S|$ and that of T is $|T|$.

  Define $a(i, j)$ to be the value of an **optimal alignment** of strings:

  $S[1], S[2], \ldots, S[i]$ and

  $T[1], T[2], \ldots\ldots, T[j]$

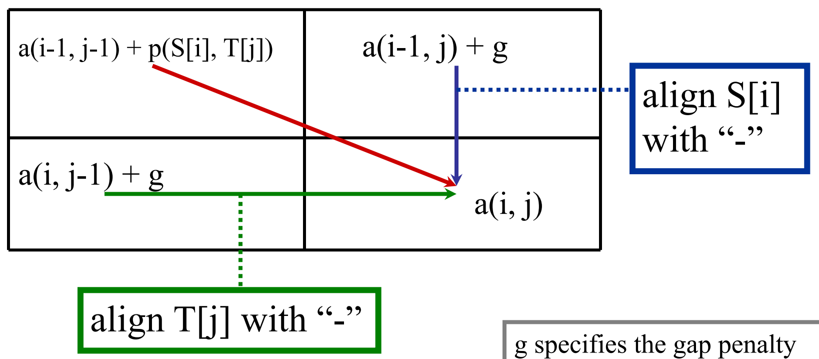
  $a(|S|,|T|)$ is the value of an optimal alignment of S and T.



Filling in the DP Table

- DP uses a table of size (|S|+1) × (|T|+1).
- $a(i, j)$ corresponds to the optimal alignment of the i^{th} prefix of S with the j^{th} prefix of T.
- The dynamic programming algorithm fills in the entries of the table (matrix) by computing the values of $a(i, j)$ from top to bottom, left to right.
- The value of the optimal alignment is given by $a(|S|,|T|)$.

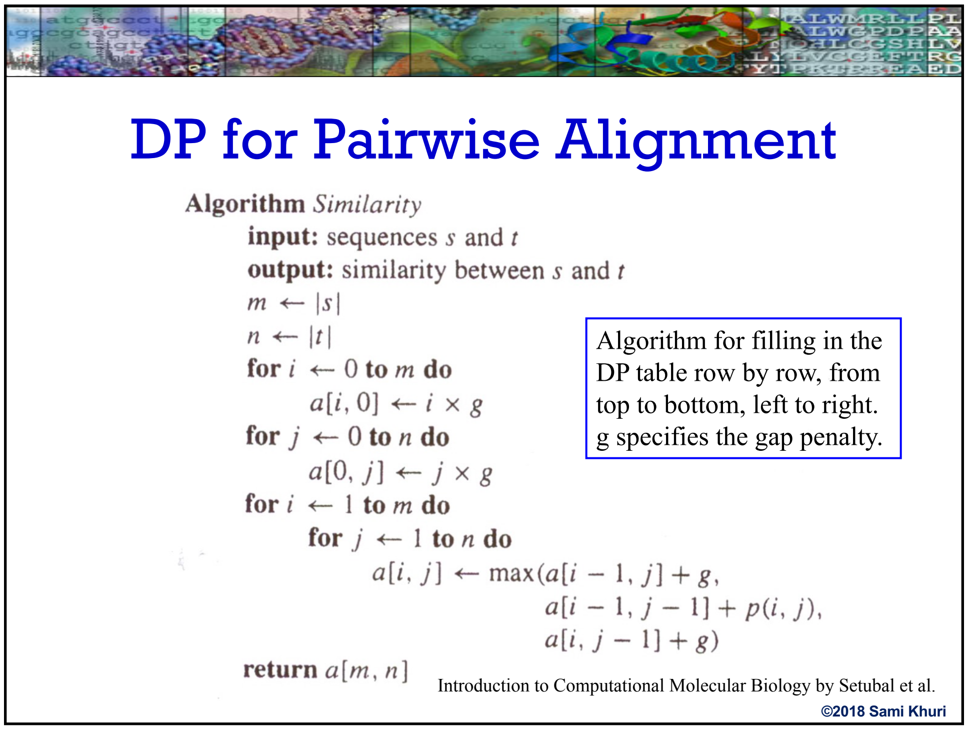
Filling Entry a(i,j) in the Table



g specifies the gap penalty
p(S[i], T[j]) = 1 if S[i] = T[j]
p(S[i], T[j]) = 0 otherwise

DP: Bookkeeping and Retracing

- Draw lines crossing the entries in the matrix to show from which entry in the matrix we derived the maximum score for each entry a(i, j).
- To determine the solution of the optimal alignment, simply retrace the steps from entry $a(|S|, |T|)$ to entry a(0, 0).



DP for Pairwise Alignment

```
Algorithm Similarity
    input: sequences $s$ and $t$
    output: similarity between $s$ and $t$
    $m \leftarrow |s|$
    $n \leftarrow |t|$
    for $i \leftarrow 0$ to $m$ do
        $a[i, 0] \leftarrow i \times g$
    for $j \leftarrow 0$ to $n$ do
        $a[0, j] \leftarrow j \times g$
    for $i \leftarrow 1$ to $m$ do
        for $j \leftarrow 1$ to $n$ do
            $a[i, j] \leftarrow \max(a[i-1, j] + g,$
                                   $a[i-1, j-1] + p(i, j),$
                                   $a[i, j-1] + g)$
    return $a[m, n]$
```

Algorithm for filling in the DP table row by row, from top to bottom, left to right. g specifies the gap penalty.

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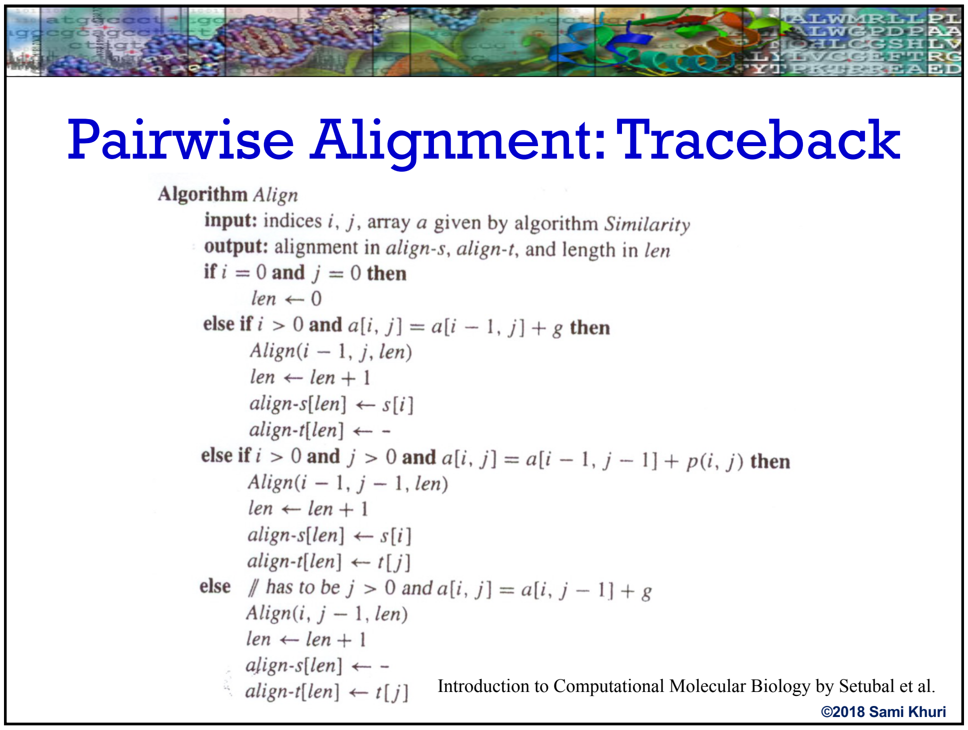

Pairwise Alignment: Traceback

```
Algorithm Align
    input: indices $i, j$, array $a$ given by algorithm Similarity
    output: alignment in align-s, align-t, and length in len
    if $i = 0$ and $j = 0$ then
        $len \leftarrow 0$
    else if $i > 0$ and $a[i, j] = a[i-1, j] + g$ then
        Align($i - 1, j, len$)
        $len \leftarrow len + 1$
        align-s[len] $\leftarrow s[i]$
        align-t[len] $\leftarrow$ -
    else if $i > 0$ and $j > 0$ and $a[i, j] = a[i-1, j-1] + p(i, j)$ then
        Align($i - 1, j - 1, len$)
        $len \leftarrow len + 1$
        align-s[len] $\leftarrow s[i]$
        align-t[len] $\leftarrow t[j]$
    else    // has to be $j > 0$ and $a[i, j] = a[i, j-1] + g$
        Align($i, j - 1, len$)
        $len \leftarrow len + 1$
        align-s[len] $\leftarrow$ -
        align-t[len] $\leftarrow t[j]$
```

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