CS 252:

Advanced Programming Language Principles

Operational Semantics, Continued

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Topics covered today

- Small-step operational semantics
- Representing mutable state
- Evaluation contexts

Review: Bool* Language

```
e ::=
    true
  | false
  | if e
    then e
    else e
```

expressions:

constant true

constant false

conditional

Big-step operational semantics evaluate every expression to a value.

The expression `e` …

$$e \Downarrow v$$

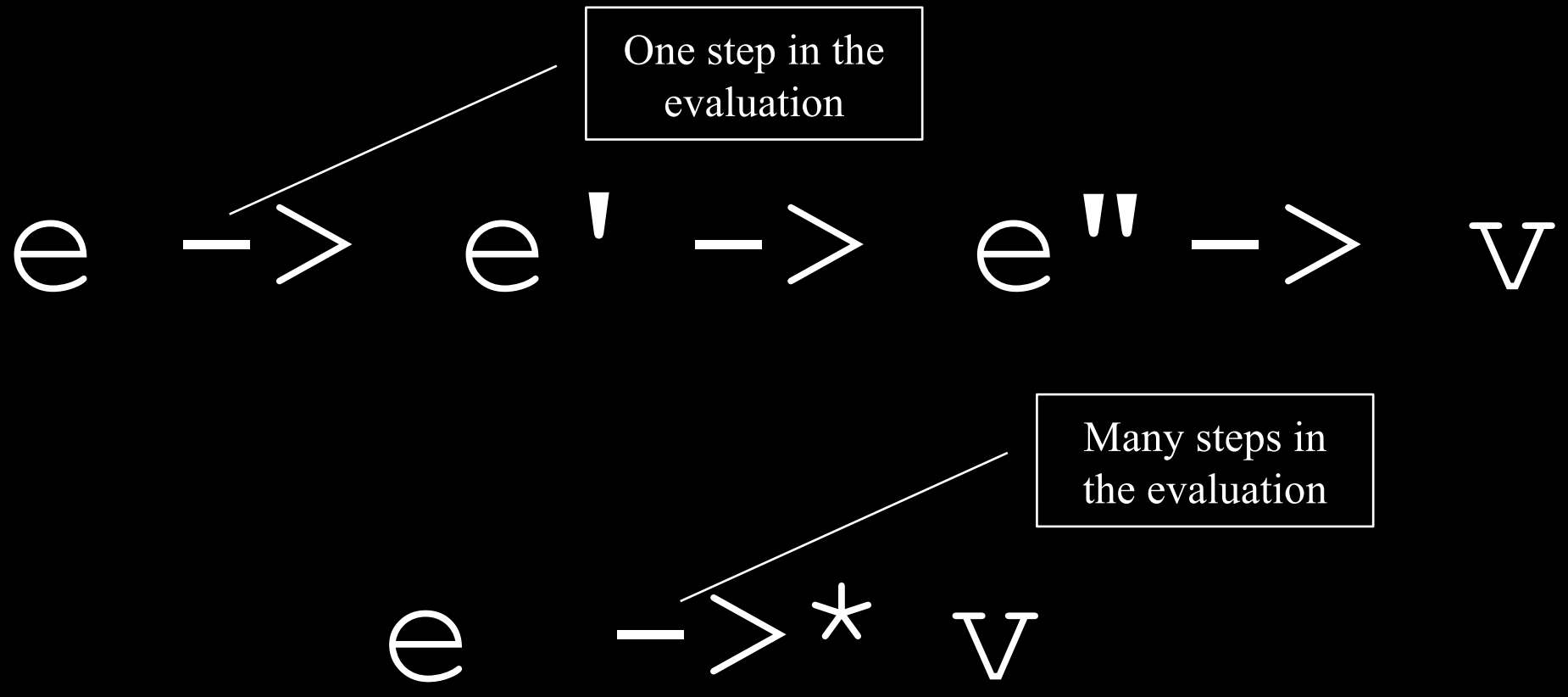
… evaluates to …

… the value `v`.

Small-step operational semantics evaluate an expression until it is in *normal form*.

"normal form" – it cannot be evaluated further.

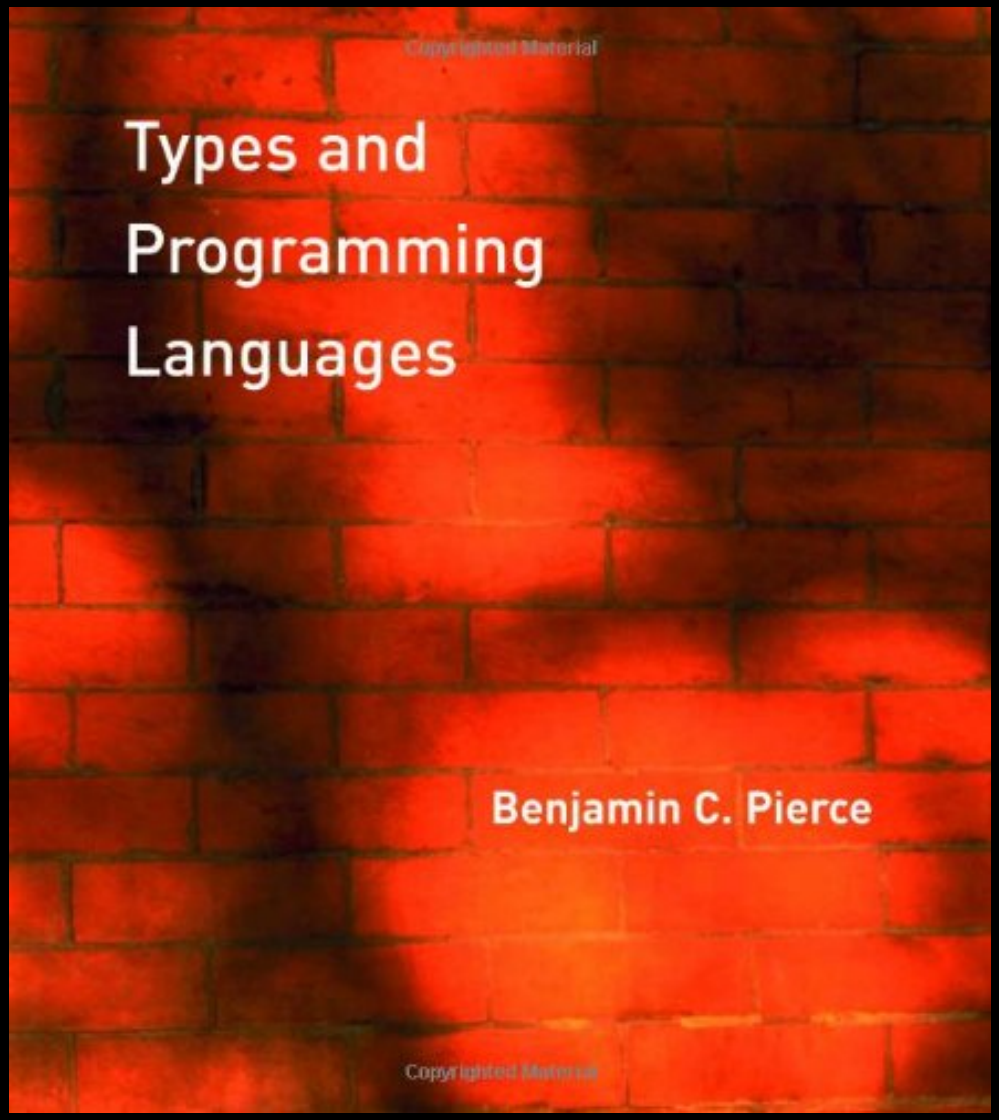
Small-Step Evaluation Relation



TAPL

The top reference for more details on PL formalisms.

Available at library.



Review: Big-step semantics for Bool*

B-IfTrue

$$\frac{e1 \Downarrow \texttt{true} \qquad e2 \Downarrow v}{\texttt{if } e1 \texttt{ then } e2 \texttt{ else } e3 \Downarrow v}$$

B-IfFalse

$$\frac{e1 \Downarrow \texttt{false} \qquad e3 \Downarrow v}{\texttt{if } e1 \texttt{ then } e2 \texttt{ else } e3 \Downarrow v}$$

B-Value

$$\frac{}{v \Downarrow v}$$

Small-step semantics for Bool*

(in-class)

Bool* Small-Step Semantics

E-IfTrue

```
if true then e2 else e3 -> e2
```

E-IfFalse

```
if false then e2 else e3 -> e3
```

E-If

$$\frac{\texttt{e1 -> e1'}}{\texttt{if e1 then e2 else e3 -> if e1' then e2 else e3}}$$

Let's develop operational semantics for the WHILE language.

Unlike Bool*, WHILE supports *mutable references*.

WHILE Language

`e ::= a`	variables/addresses
`| v`	values
`| a:=e`	assignment
`| e;e`	sequence
`| e op e`	binary operations
`| if e then e else e`	conditionals
`| while (e) e`	while loops

WHILE Language (continued)

```
v  ::=  i          integers
     |  b          booleans

op  ::=  +   |  -    binary operators
       |  \   |  *
       |  <   |  >
       |  <=  |  >=
```

Small-step semantics with state

Since Bool* did not have mutable references, our evaluation rules only handled expressions:

E-IfFalse

```
if false then e2 else e3 -> e3
```

WHILE *does* allow for imperative updates, so we need to modify our semantics.

Bool* vs. WHILE evaluation relation

Bool* relation:

$$e \rightarrow e'$$

WHILE relation:

$$e, \sigma \rightarrow e', \sigma'$$

A "store", represented by the Greek letter sigma

Important Note on The Store

- We can represent state in big-step operational semantics with the same notation.
- Small-step relation for WHILE language:
  `e,σ -> e',σ'`
- Big-step relation for WHILE language:
  `e,σ ⇓ e',σ'`

The Store

- Maps *references* to values
- Some key operations:
  - `σ(a)`: Get value at "address" `a`
  - `σ[a:=v]`: New store identical to `σ`, except that the value at address `a` is `v`.

In-class: Specify semantics for the WHILE language (`e,σ -> e',σ'`)

`e ::= a`	variables/addresses
`| v`	values
`| a:=e`	assignment
`| e;e`	sequence
`| e op e`	binary operations
`| if e then e else e`	conditionals
`| while (e) e`	while loops

Evaluation order rules specify an order for evaluating expressions.

Reduction rules rewrite the expression.

E-IfFalse (reduction)

```
if false
  then e2
  else e3 -> e3
```

E-If (evaluation order)

$$\frac{\texttt{e1 -> e1'}}{\texttt{if e1 then e2 else e3 -> if e1' then e2 else e3}}$$

Concise representation of evaluation order rules

- Evaluation order rules tend to
  - be repetitive
  - clutter the semantics
- Evaluation contexts represent the same information concisely

A *redex* (reducible expression) is an expression that can be transformed in one step

Which expression is a redex?

1. `if true then(if true then false else false) else true`

2. `if (if true then false else false) then false else true`

The whole expression is a redex: a rule transforms "if true …"

Condition needs to be evaluated first: not a redex

Evaluation Contexts

- Replace evaluation order rules
- Marker (•) or "hole" indicates the next place for evaluation.
  - C = `if • then true else false`
  - r = `if true then true else false`
  - C[r] = `if (if true then true else false) then true else false`

The original expression

Rewriting our evaluation rules

The rules now apply to a redex *within the specified context.*

Note the addition of the C[…] to the rule

EC-IfFalse

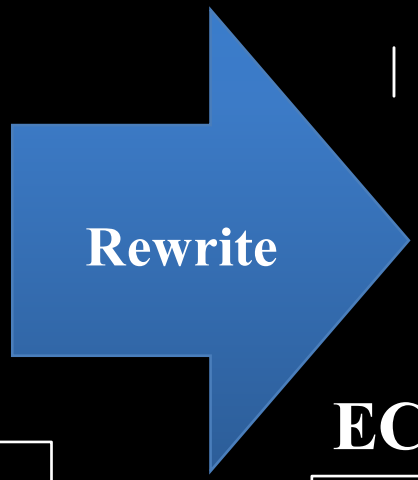
```
C[if false
    then e2
    else e3] -> C[e3]
```

E-If (evaluation order)

$$\frac{e1 \rightarrow e1'}{\texttt{if e1 then e2 else e3} \rightarrow \texttt{if e1' then e2 else e3}}$$

Context:

```
C ::= •
  | if C then e
      else e
  | …
```



E-IfFalse (reduction)

```
if false
  then e2
  else e3 -> e3
```

EC-IfFalse

```
C[if false
 then e2
 else e3]->C[e3]
```

In class: let's rewrite our evaluation rules in the new format.

Homework #2: WHILE Interpreter

- Part 1: Rewrite the semantics for WHILE to use big-step semantics.
- Part 2: Write an interpreter for WHILE. Starter code is available on the course website

Haskell does not have mutable state.
How can we write a program that does?

Introducing `Data.Map`...

Data.Map

- Maps are **immutable**.
- Useful methods:
  - `empty`: creates a new, empty map
  - `insert k v m`: returns a new, updated map
  - `lookup k m`: returns the value for key `k` stored in map `m`, wrapped in a `Maybe` type
- See "Learn You a Haskell", Chapter 7