

This paper proves two combinatorial principles related to playing a fixed number of two person games in parallel. The first principle says there is a two person infinite game G and two strategies f, g such that if G is played on two boards and Player 1 plays according to f then Player 1 is guaranteed to win at least once and if Player 2 plays according to g then Player 2 is guaranteed to win at least once. The second principle concerns two rounds of two person infinite games played on k boards where the two players exchange roles after each round. Strategies in this setting consists of a pair of strategies (h, H) , one for the first round and one for the second. The second round can be based on a knowledge of all that occurred in the first round. The principle states that there are games G_1, \dots, G_4 and a strategy (h, H) for Player 1 that guarantees a victory twice on at least one of the four boards. One can also show no such games exist for fewer than four boards. These two principles are related in that the paper goes on to show that if one takes the games G_1, \dots, G_4 and a strategy (h, H) satisfying the second principle, then for each G_i one can define from this information strategies f_i and g_i such that f_i, g_i and G_i satisfy the first principle.

The author states that the motivation for these principles comes from trying to exhibit principles which are unprovable in $I\Delta_0 + \Omega_1$. This is the fragment of Peano Arithmetic with induction only for bounded formulas and with an axiom saying $x^{\lceil \log(x+1) \rceil}$ is total. Its study is closely connected to open questions in computational complexity theory. The author formulates polynomial time variants of his two principles which he conjectures are not provable in $I\Delta_0 + \Omega_1$. His evidence to support his conjecture is the results about the infinite principles and the fact that the second principle came from the study of the Herbrand consistency of $I\Delta_0 + \Omega_1$. Although it seems unlikely that current techniques can yield such independence results, the author suggests several interesting more tractable problems based on his polynomial variants.