In classical logic it is known that the theory S^1_2 can be axiomatized over the base theory $BASIC$ using either Π^b_1-$PIND$ or Σ^b_1-$PIND$ [1]. Here Π^b_1 formula correspond to $coNP$ predicates and the Σ^b_1-formulas correspond to the NP predicates. $PIND$ is a "polynomial" induction schema. This paper shows via a model theoretic proof that if one considers intuitionistic theories the analogous result does not hold.

The papers defines a Kripke model to be T-normal for some theory T if each of its worlds satisfies T. Given two classical models M, N of $BASIC$, the model M is said to be a *weak end extension* of N, if M extends N and if its elements which can be bounded by a length of some element in the model extend the corresponding elements in N. This notion is meaningful in models of weak arithmetic since in general exponentiation is not total. For the intuitionistic case we say K' is a weak end extension of K if each of its worlds weak end extends a corresponding world in K. One of the papers main results is that: Any reversely well-founded $BASIC$-normal weak end extension Kripke model whose terminal worlds model S^1_2 forces $BASIC$+Π^{b+}_1-$PIND$. The plus in Π^{b+}_1 is used to denote Π^b_1-formulas not containing negation or implication. By a result of Johannsen [2] there is a model of S^1_2 which has a submodel M' that weak end extends to M such that limited subtraction is not total in M'. Since S^1_2 can define limited subtraction and is $\forall\Sigma^b_1$-conservative over IS^1_2 (intuistionistic S^1_2), this shows $BASIC$+Π^{b+}_1-$PIND$ does not imply IS^1_2.

The second main result of the paper is that the union of the worlds in any linear weak end extension Kripke model of $BASIC$+Π^{b+}_1-$PIND$ satisfies $BASIC$+Π^{b+}_1-$PIND$. Using this result the paper shows that if IPV, the intuitionistic theory of PV plus polynomial induction on NP formulas, proves $coNP$-$PIND$ then the classical closure of of IPV, CPV, is equal to PV_1. By a result of Krajíček, Pudlák, and Takeuti this is known to imply the collapse of the polynomial hierarchy. A corollary of this is that IS^1_2 does not prove Π^{b+}_1-$PIND$ unless the polynomial hierarchy collapses.

The paper is well presented with clear and short proofs.

References

[1] S.R. Buss. Bounded Arithmetic. Bibliopolis. 1986.

[2] J. Johannsen. A remark on independence results for sharply bounded arithmetic. Mathematical Logic Quarterly. Vol 44. pp 569–570. 1998.

[3] J. Krajíček, P. Pudlák, and G. Takeuti. Bounded arithmetic and the polynomial hierarchy. Annals of Pure and Applied Logic. Vol 52. pp143–153. 1991.