This paper considers tableau based variations of the Davis Putnam Logemann Loveland (DPLL) procedure [2][1]. The usual DPLL procedure is used to check satisfiability of conjunctive normal form (CNF) formulas; whereas, this paper's methods apply directly to Boolean circuits. The main system developed, BC, works for AND, OR, NOT circuits. It has an explicit cut rule and seven rules for standard Boolean constraint propagation. One factor which affects the efficiency of DPLL is the choice of clause to split/cut upon. Motivated by heuristics which have been tried in practice, the paper explores restrictions on the kinds of cuts allowed in tableaux. The variations considered are: (1) to only allow cuts on input gates (BC_i), (2) to only allow cuts on input gates or on bottom-up gates v – these latter must have some child v' that has already been given a value Tv' of Fv' in that branch of the tableau, (3) to only allow cuts on the output gate or on top-down gates v – top-down gates must have some parent v' in the current branch which has an entry Tv' or Fv', (4) to only allow cuts on input or top-down gates (BC_{i+td}), and (5) to only allow cuts on bottom-up or top-down gates (BC_{bu+td}). The paper uses clever constructions and the known hardness of the pigeonhole principle [3] and of an XOR_n gadget to show exponential separations between these variations. In addition to the inclusions they show, these separations give the strict hierarchies $\mathsf{BC} \succ \mathsf{BC}_{bu+td} \succ \mathsf{BC}_{bu} \succ \mathsf{BC}_i$ and $\mathsf{BC} \succ \mathsf{BC}_{bu+td} \succ \mathsf{BC}_{i+td} \succ \mathsf{BC}_{td}$. It is also established that BC_{bu} and BC_{i+td} are incomparable and that BC_{td} and BC_i are incomparable. In the last section of the paper, it is shown that this paper's general tableau setting for circuits, BC, is polynomial equivalent to DPLL for CNFs that come from circuits via the standard Tseitin translation of introducing new variables to convert a circuit to an equivalent CNF.

References

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