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Quantum Branching Programs

On the Power of Classical and

Moore

Joint work with P. Ablavsky, A. Carheels, M. Karlin, and C.
Outline

- Power of these models
- Making things random or quantum
- Barrington's result
- An algebraic definition
- Uses of branching programs
- What are branching programs?
More about the model

- Allowed to query same variable more than once
- For this talk can break graph into levels according to distance from source.
- Width of program is number of nodes at a level.
- Look at families of programs \{F_n\} such that
- \( F_n \) computes a given function on \( n \) variable inputs.
Intuitions
Uses of branching programs

- Counting problems, genetic programming
- Other uses: test generation, network flow
- Can also use to verify sequential circuits
- Restricted type and verify their equivalence
- Corresponding BPs for each of the above
- Suppose we can get a function f and a circuit R
What are branching programs?

Acyclic graphs

Edges labelled with variables

Follows paths from source to a sink

Read off value of sink

AND Function

x₁
x₂

0

x₁
-x₁

x₂
- x₂

x₂, - x₂
An algebraic definition

depending on the two matrices multiplying one of the next as from one level to operation of going Can view the value of a variable.
Barth's argument.

necessary properties to allow one to do

dodecahedron. The group of these symmetries has
dimensions. So can do rotations for symmetries of
matrices can be used to represent rotations in 3

The NC\(^1\) result uses the fact that 2 x 2 unitary
quantum and random programs for Mod 8 gates.
Can also show a gap on read-once models between

More Ideas
The width of program correspond to the number of rows or columns in the matrix. 

The variables and seeing what if the final state. 

To evaluate a branching program can thus: More on algebraic view.
Barrington’s Result

- Barrington ’86 used this algebraic idea, together with the fact you can come up with 5 \times 5-matrices A, B such that \( ABA^{-1}B^{-1} \) is not the identity matrix in a special way to show width-5 branching programs can simulate log-depth, polynomial size AND, OR, NOT – circuits. His result \( 5\text{-BP} = \text{NC}^1 \).
Making things random or quantum

- Algebraic point of view makes it easy to define randomized or quantum programs.
- In random case, we allow entries in matrices to be from interval \([0, 1]\) and such that the rows sum to 1.
- In quantum case, we take matrices \(U\) over complex numbers such \(U^†U=I\). (Unitary).
- Width can be defined in terms of matrix size.
- In both case need to define what it means to accept in terms of probability see a 1 after performing the matrix multiplications on an input vector.
Comprehend exactly NC1.

Width-2 quantum branching programs

Majority with success \( \geq 3/4 \).

Randomized model above (can recognize NO width-2 stochastic program (our)

Power of these models
between accepting and rejecting states. And that we need to have a certain distance nor quantum matrices can increase distances. Result exploits the fact that neither the stochastic with.

In the case where we get on acceptance error versus program trade-off, result in both the quantum and random. The first result actually follows from a general.

Ideas behind these results