This paper works out in detail an interpretation of Buss' theory S^1_2, [1], into the Ferreira's theory Σ^{b}_1-NIA, [2]. This correspondence had been mentioned in several earlier works, but had never previously been formally carried out. The authors carefully define both theories. The arithmetic theory S^1_2 consists of thirty two axioms for the symbols 0, S, $+$, $\cdot$, $|\cdot|$ (length of x), $\lfloor \frac{1}{2} \cdot \rfloor$, $\#$ ($x \# y := 2^{|x||y|}$), and $\leq$, together with the PIND axioms:

$$A(0) \wedge \forall x(A(\lfloor \frac{1}{2} x \rfloor) \rightarrow A(x)) \rightarrow \forall x A(x)$$

for Σ^b_1-formulas. Here Σ^b_1-formulas are the closure under $\vee$, $\wedge$, bounded existentiation, and length bounded quantifications of the class of formulas which have all their quantifiers bounds by the length of some term. The string theory Σ^{b}_1-NIA consists of fourteen axioms for the symbols ϵ, 0, 1, $\frown$ (concatenation), $\times$ ($x \times y :=$ concatenate x to itself $|y|$-times), $\subseteq$ (initial substring), together with induction on notation axioms:

$$B(\epsilon) \wedge \forall x(B(x) \rightarrow B(x0) \wedge B(x1)) \rightarrow \forall x B(x)$$

for Σ^b_1-formulas. In the context of the theory Σ^{b}_1-NIA, the Σ^{b}_1-formulas are the closure under $\vee$, $\wedge$, bounded quantifiers of the form $\exists x(1 \times x \subseteq 1 \times t)$, and subword quantifications of the class of formulas all of whose quantifiers are subword quantifiers. After introducing these theories, the authors briefly review the definition of interpretation as presented in Enderton [3]. The interpretation from S^1_2 into Σ^{b}_1-NIA roughly involves interpreting 0 as the empty string, interpreting natural numbers as either the empty strings or strings starting with a 1, and coming up with appropriate mappings of each of the function and relation symbols of S^1_2. The paper then shows how Σ^{b}_1-NIA can prove the translations of each of S^1_2's thirty-two BASIC axioms as well as its induction scheme. The paper is very clear and well-written.

References

[1] S. Buss. Bounded Arithmetic. Bibliopolis, Napoli (1986).

[2] F. Ferreira. Polynomial Time Computable Arithmetic and Conservative Extensions. Ph.D. Dissertation, Pennsylvania State University (1988).

[3] H. Enderton. A Mathematical Introduction To Logic. Academic Press (1991).