

$$\text{distance} = \frac{1}{\|2, 3\|} = \frac{1}{\sqrt{13}}$$

let $x = (2, 3)$

for $x \cdot y = 1$

$y = (-1, 1)$
 $(-2, 2)$

$x \cdot y = 2$

$y = (1, 0)$
 $(2, -2)$
 $(-5, 4)$

$(0, -6)$
 $(1, -7)$
 $(2, -8)$
 $(3, -9)$
 $(4, -10)$

$x \cdot y = 3$

$y = (0, 1)$
 $(-6, +5)$

$x \cdot y = 4$

$y = (2, 0)$
 $(-1, +2)$
 $(7, +6)$
 $(-4, +4)$

$$\text{distance} = \frac{1}{\|1, 1\|} = \frac{1}{\sqrt{2}}$$

let $x = (1, 1)$

for $x \cdot y = 1$

$y = (0, 1)$
 $(1, 0)$
 $(-1, 2)$
 $(-2, 3)$
 $(-3, 4)$
 $(-4, 5)$

$(2, -1)$
 $(3, -2)$
 $(4, -3)$

$x \cdot y = 2$

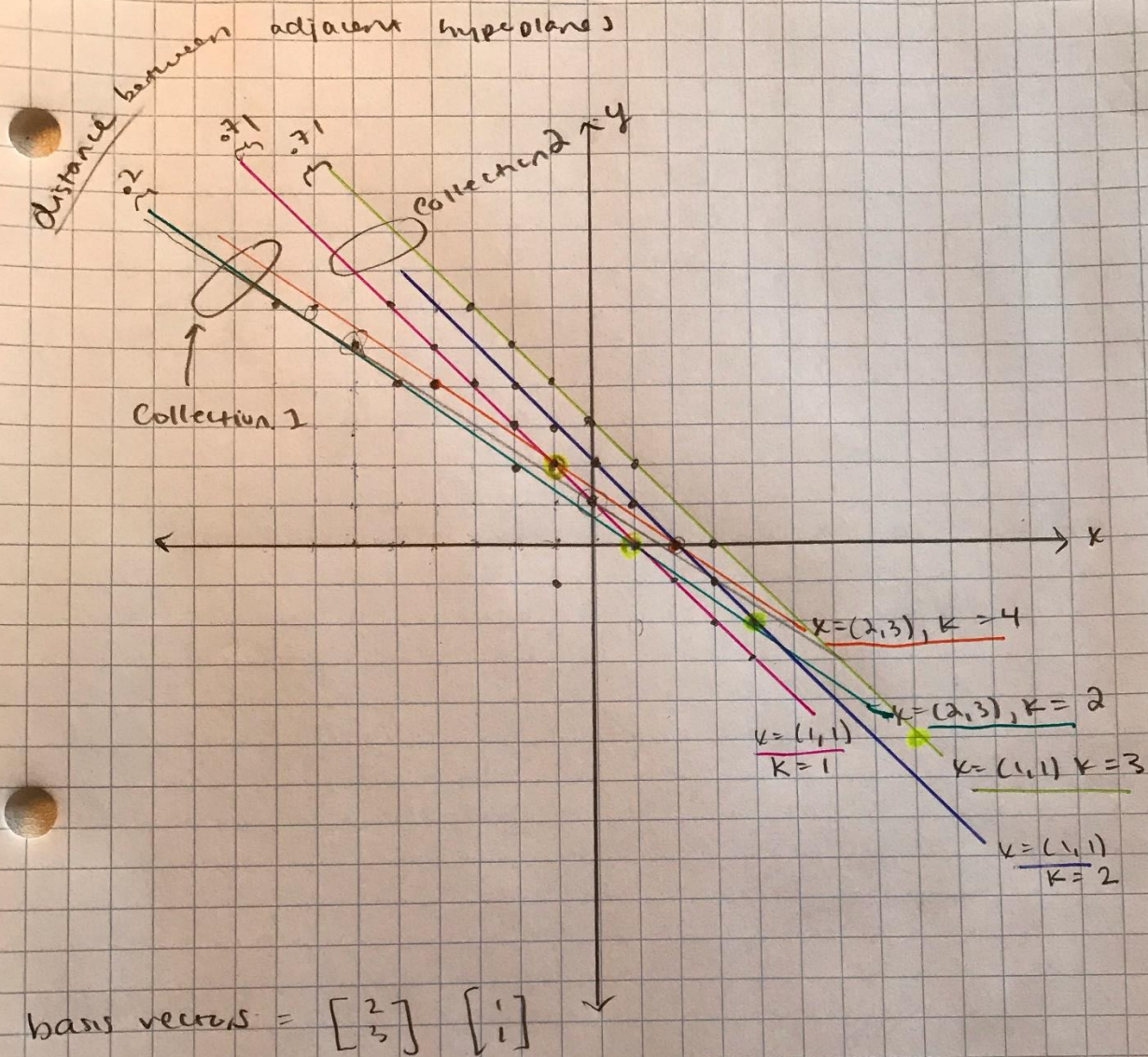
$y = (1, 1)$
 $(2, 0)$
 $(0, 2)$
 $(1, 1)$
 $(-1, -3)$
 $(2, -4)$
 $(3, -6)$

$(3, -1)$
 $(4, -2)$

$x \cdot y = 3$

$y = (2, 1)$
 $(1, 2)$
 $(3, 0)$
 $(0, 3)$
 $(-1, 4)$
 $(-2, 5)$
 $(-3, 6)$

$(4, -1)$
 $(5, -2)$
 $(6, -3)$



$n = \# \text{ of basis vectors} = 2$

• The dual lattice is the set of points that are intersections of $n=2$ hyperplanes, one from each collection

• This shows that every point in the dual lattice has an integer value when multiplied by a basis vector

• Since every point in the original lattice can be represented as a linear combination of the basis vectors, every point in the dual lattice has an integer value when multiplied by any vector from the original lattice