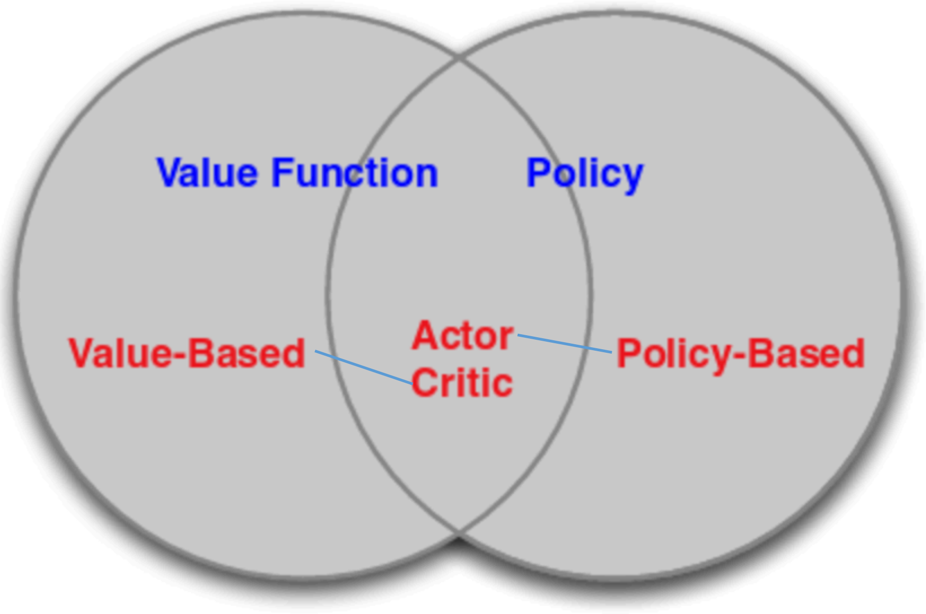
Deep Deterministic Policy Gradient (DDPG)

Actor-Critic Approach



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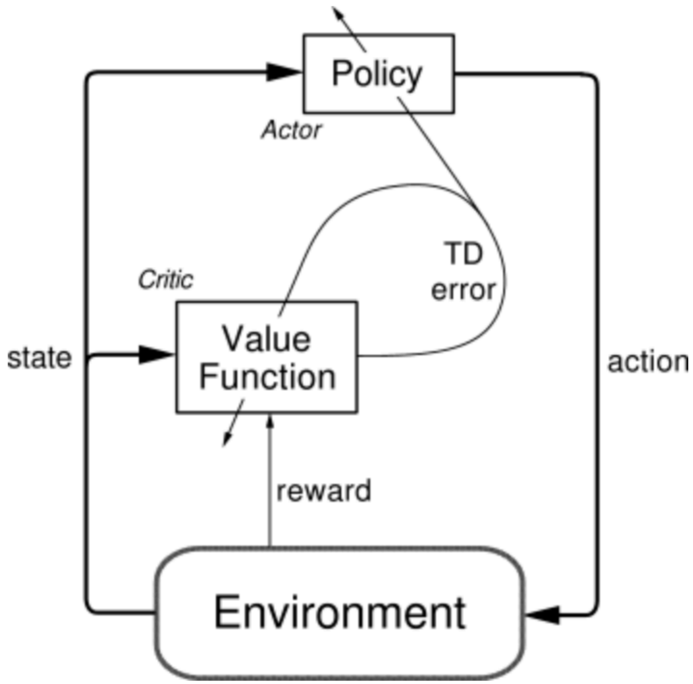

- Actor function $\mu(s|\theta^{\mu})$ specifies action *a* given the current state of the environment *s*
- Critic value function $Q(s, a|\theta^{Q})$ specifies a signal (TD Error) to criticize the actions made by the actor

Deterministic vs Stochastic Policy

- Stochastic : $\pi_\theta(a|s) = \mathbb{P}[a|s;\theta]$
- Deterministic : $a = \mu_\theta(s)$
- Computing stochastic gradient requires more samples, as it integrates over both state and action space. Deterministic gradient is preferable as it integrates over state space only.
- In DQN, action was selected as:

$$a_t = \max_a Q^*(\phi(s_t), a; \theta)$$

- Above eq. is not practical for continuous action space. Using deterministic policy allows us to use:

$$a_t = \mu(s_t|\theta^\mu)$$

Deterministic vs Stochastic Policy

- But, deterministic policy gradient might not explore the full state and action space. To overcome this, we introduce a noise process $\mathcal{N}$:

$$a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$$

- Recall that we used ϵ - greedy approach in DQN to ensure exploration.

Deterministic Policy gradient

- We want to maximize the rewards (Q-values) received over the sampled mini-batch. The gradient is given as:

J is the start distribution

$$\nabla_{\theta^\mu} J \approx \mathbb{E}_{s_t \sim \rho^\beta} \left[\nabla_{\theta^\mu} Q(s, a|\theta^Q)|_{s=s_t, a=\mu(s_t|\theta^\mu)} \right]$$

- Applying chain rule:

$$= \mathbb{E}_{s_t \sim \rho^\beta} \left[\nabla_a Q(s, a|\theta^Q)|_{s=s_t, a=\mu(s_t)} \nabla_{\theta_\mu} \mu(s|\theta^\mu)|_{s=s_t} \right]$$

- Silver el at. (2014) proved that this is the policy gradient, i.e. we will get the maximum expected reward as long as we update the model parameters following the gradient formula above.

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
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Use target networks to do off-policy updates

Action selected using Deterministic Actor as explained before

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    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R — Experience Replay
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
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Experience Replay

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    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$

    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

Policy Gradient

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— Policy Gradient

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— τ << 1
Slow updates,
but highly stable

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