Solving Square Puzzles – Base Approach

Literature analysis of an approach by Tal and Paikin to solve square jigsaw puzzle (with my observations for shredded documents in red)

Square puzzle challenges

- Choosing the first piece
- Puzzle has missing pieces
- Unknown board size
- Rotatable parts (Shredded documents have 2 rotational states totally)
- Bigger picture not known (Expected final image)
- Mixed bag of puzzles

Key terms

- **Dissimilarity between pieces:** The last 2 pixels of a piece in each row (or column) at the boundary is used to predict the first pixel of the neighboring piece. Challenge is harder for vertical shredding and documents with texts instead of images.
- **Compatibility:** The likelihood that 2 pieces are neighbors. C(Pi,Pj,r) is a function that calculates the likelihood that pieces Pi and Pj are neighbors in the placements r Ɛ {Up,Down,Left,Right}.
- **Reliability:** The robustness of dissimilarity metric to identify neighboring pieces.
- **Best buddies:** Both pieces identify each other as the most likely neighbor.

Phases of the solver

- Estimate compatibility between pieces
- Identify the first piece to begin solving puzzle
- Piece placement

Dissimilarity Metric

$$D(p_i, p_j, right) = \sum_{k=1}^{K} \sum_{d=1}^{3}$$
$$[([2p_i(k,K,d) - p_i(k,K-1,d)] - p_j(k,1,d))^p +$$
$$([2p_j(k,1,d) - p_j(k,2,d)] - p_i(k,K,d))^p]^{q/p}.$$

Where D(Pi,Pj,right) indicated the dissimilarity between piece i and piece j with piece j to the right of piece I.

K is the piece size and also the last pixel and d is the LAB color space dimension.

The applicability of LAB color space dimension for documents needs to be researched more.

Cntd..

- The previous equation was from Pomeranz et al. which was in L 1/16 3/10 norm. This caused large differences to be considered small ones.
- L2 norm on the other hand exaggerates even small differences.
- In this paper, they reached a compromise and used L1 norm.
- This improved the speed of computation.
- **Asymmetric dissimilarity** was used where

$$D(p\_i, p\_j, right) \neq D(p\_j, p\_i, left)).$$

The use of asymmetric dissimilarity helps when there are missing pieces.

$$D(p_i, p_j, right) = \sum_{k=1}^{K} \sum_{d=1}^{3}$$

$$\|([2p_i(k, K, d) - p_i(k, K-1, d)] - p_j(k, 1, d))\|$$

Compatibility Function

- Establishes reliability that a small dissimilarity between pieces indicates true adjacency and the value does not correspond to smooth areas which make for weak initial pieces.

$$C(p_i, p_j, r) \quad = \quad 1 \quad - \quad \frac{D(p_i, p_j, r)}{secondD(p_i, r)}$$

The higher the C value, the better it is to use piece Pi as the best piece for placement.

Best Buddies metric

$$\forall p_k \neq p_j, C(p_i, p_j, r_1) \geq C(p_i, p_k, r_1)$$

$$\forall p_k \neq p_i, C(p_j, p_i, r_2) \geq C(p_j, p_k, r_2).$$

The opposing relation for r1="left" is r2="right" and vice versa.
The opposing relation for r1="up" is r2="down" and vice versa.

Initial Piece Placement

- The first piece should have best buddies in all r {left,right,up,down}.
- Each neighbor of the piece should have best buddies in all r {left,right,up,down}.
- Select a piece that maximizes the mutual compatibility score.

$$\tilde{C}(p_i, p_j, r_1) = \tilde{C}(p_j, p_i, r_2) = \frac{C(p_i, p_j, r_1) + C(p_j, p_i, r_2)}{2}$$

Placement

Algorithm 1 Placer

```
While there are unplaced pieces
    if the pool is not empty
        Extract the best candidate from the pool
    else
        Recalculate the compatibility function
        Find the best neighbors (not best buddies)
    Place the above best piece
    Add the best buddies of this piece to the pool
```

- The best candidate is the one with the highest mutual compatibility value.
- If the pool is exhausted, compatibility function is recalculated and best neighbors are chosen.
- The criteria has been relaxed a little bit by allowing best neighbors instead of best buddies.
- This is akin to how humans solve puzzles. When a stage is reached where we cannot solve further, we start anew with remaining pieces and relax our match criteria.
- Mixed puzzles are solved by fixing a threshold value (0.5 in the paper) for compatibility between placed and unplaced pieces. When all the placed pieces have compatibility scores lower than 0.5 with unplaced pieces, a new puzzle is started.

Areas requiring attention in shredded documents reconstruction

- Dissimilarity metric using LAB color spaces.
- The primary limitation of the algorithm proposed in this paper is that it may not work as effectively when images are smooth or noisy. Documents in general have very small dissimilarity measure making them smooth.
- Accounting for rotation.