(1) There is a cost associated with hiring a new person, $h_c + i_c$, h_c = hiring cost, i_c = interviewing cost.

Each time a candidate comes along that is "better" than all previous candidates. that candidate is hired.

The worst case scenario is when every candidate interviewed is better than all the previous candidates. This is the case when the order of the candidates is in ascending ~~and~~ rank order.

The worst case in the hiring problem is $O(n h_c + n i_c)$ because we must hire every candidate.

(2) A distribution $Pr\{\}$ on a sample space S is a mapping from events of S to real numbers such that the following axioms are satisfied

(a) $Pr\{A\} \geq 0$ for any event A

(b) $Pr\{S\} = 1$

(c) $Pr\{A \cup B\} = Pr\{A\} + Pr\{B\}$ for any two mutually exclusive events

A random variable X is a $f^{\underline{n}}$ from a sample space S to the real numbers.

The indicator random variable $I\{A\}$ associated with event A is defined as

$$I\{A\} = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{if } A \text{ does not occur.} \end{cases}$$

Such variables allow one to convert questions about probabilities to ones about expectations as they have the property that:

$$E[I\{A\}] = Pr\{A\}.$$

Refer to:

(3). Slide #6 of Feb 1st's note:

proof:

$Pr\{X_1 \cap X_2 \cap \dots \cap X_n\} = Pr\{X_1\} Pr\{X_2 | X_1\} \dots Pr\{X_n | X_{n-1} \cap \dots \cap X_1\} = \frac{n^2}{n^2} \cdot \frac{n^2-1}{n^2} \dots \frac{n^2-(n-1)}{n^2} \geq \frac{n^2-n}{n^2} \cdot \frac{n^2-n}{n^2} \dots \frac{n^2-n}{n^2}$

$= (1-\frac{1}{n})^{n-1} \geq 1 - \frac{n-1}{n}$ since $(1-a)(1-b) > (1-a-b)$ if and nonnegative $= \frac{1}{n}$

$\therefore$ for Random$(1, n^2)$, the Probability that all the priorities is unique is at least $\frac{1}{n}$.

$\therefore$ The probability two are the same is $1-\frac{1}{n}$

proof: Random$(1, n^4)$

$Pr\{X_1 \cap X_2 \cap \dots \cap X_n\} = Pr\{X_1\} \cdot Pr\{X_2 | X_1\} \dots Pr\{X_n | X_{n-1} \cap \dots \cap X_1\}$

$= \frac{n^4}{n^4} \frac{n^4-1}{n^4} \dots \frac{n^4-(n-1)}{n^4} \geq \frac{n^4-n}{n^4} \cdot \frac{n^4-n}{n^4} \dots \frac{n^4-n}{n^4} = (1-\frac{1}{n^3})^n$

$\geq 1 - \frac{n-1}{n^3}$ since $(1-a)(1-b) > (1-a-b)$ if and nonnegative

$> 1 - \frac{1}{n^2}$

$\therefore$ for Random $(1, n^3)$, the Probability that all the priorities is unique is at least $1 - 1/n^2$

So probability two the same $\leq \frac{1}{n^2}$

(4) Let n = number of days in the year
Let K be the number of people in the room.
Let X_{ijm} be the indicator random variable

$$I\{\text{person } i, j, m \text{ have the same birthday}\}$$
$$= \begin{cases} 1 & \text{if } i, j, m \text{ have same birthday} \\ 0 & \text{otherwise.} \end{cases}$$

$$E[X_{ijm}] = \Pr\{\text{person } i, j, m \text{ have the same birthday}\}$$
$$= \frac{1}{n^2}$$

$$\text{Let } X = \sum_{i=1}^{K} \sum_{j=i+1}^{K} \sum_{m=j+1}^{K} X_{ijm}$$

linearity of expectations

$$E[X] = E\left[\sum_{i=1}^{K} \sum_{j=i+1}^{K} \sum_{m=j+1}^{K} X_{ijm}\right] = \sum_{i=1}^{K} \sum_{j=i+1}^{K} \sum_{m=j+1}^{K} E[X_{ijm}]$$

$$= \binom{K}{3} \frac{1}{n^2}$$

$$= \frac{K(K-1)(K-2)}{3!} \frac{1}{n^2} = \frac{1}{6} \underbrace{\frac{K(K-1)(K-2)}{n^2}}_{\text{want this } \geq 1}$$

So need $K(K-1)(K-2) \geq 6 \cdot 365^2$

This inequality certainly holds if

$$(K-2)^3 \geq 6 \cdot 365^2$$

So could take

$$K \geq \sqrt[3]{6 \cdot (365)^2} + 2$$

$$\boxed{K \geq 95}$$

note this is within 2 + roundoff of the exact answer

#5

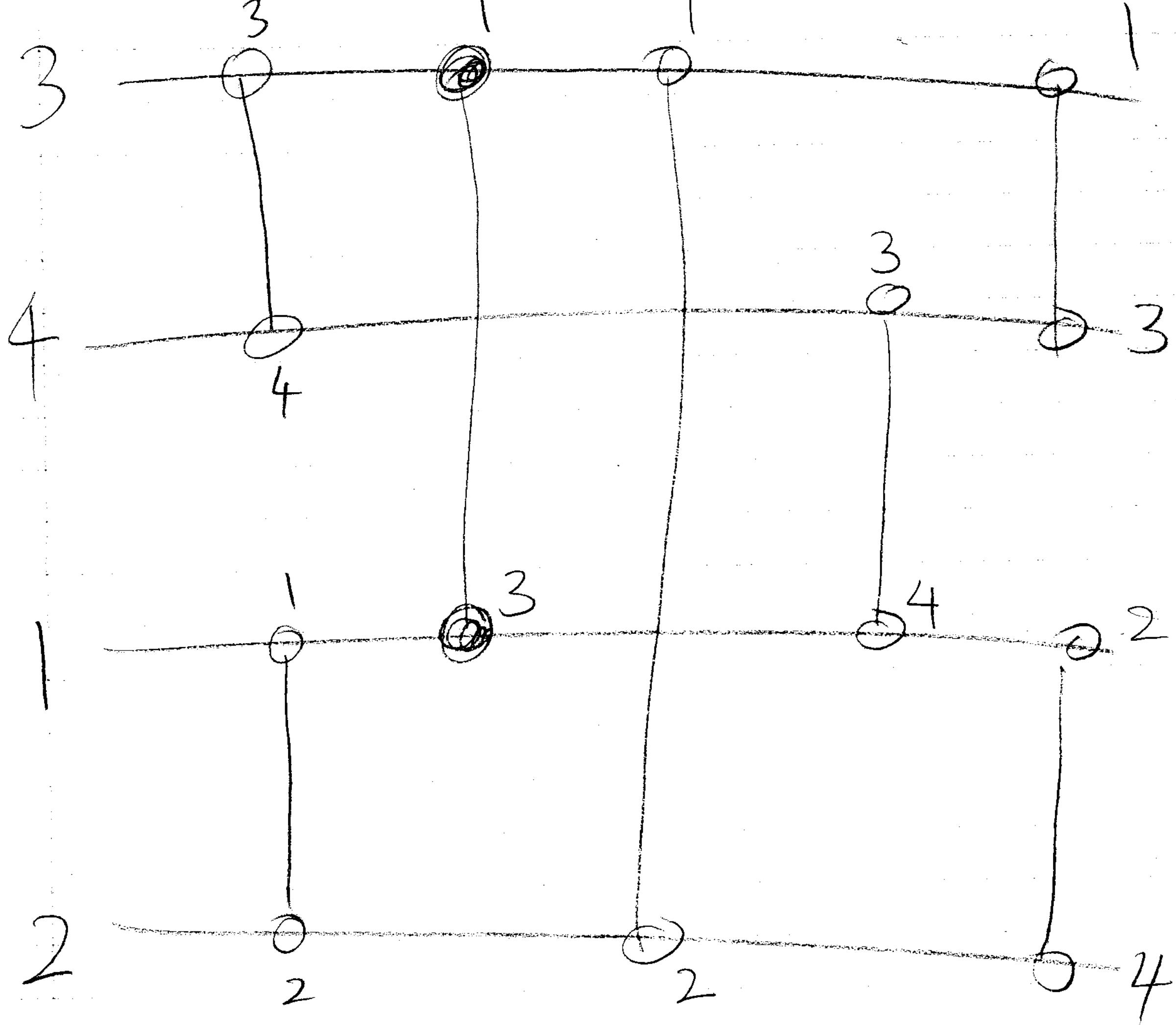
This problem is same as Coupon Collector Problem

Solution is $n \ln n$

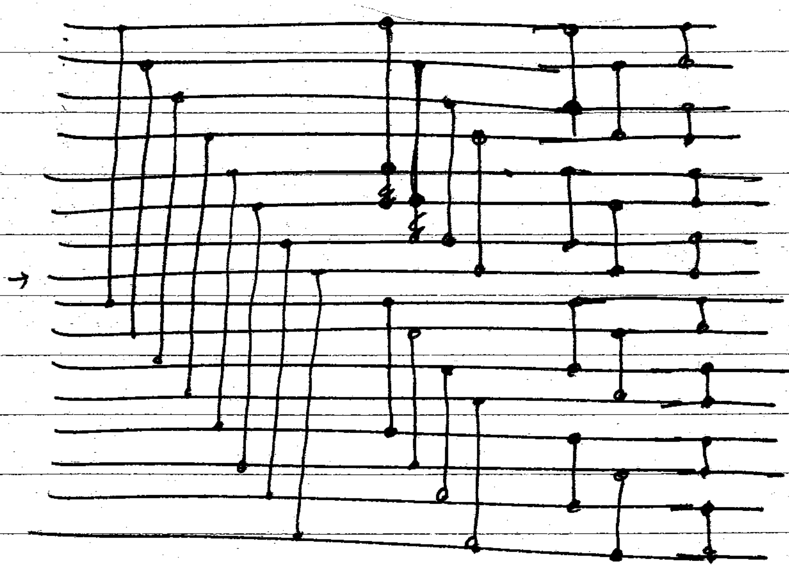
Here $n=10$

So, the Solution is $\underline{\underline{10 \ln 10}}$

6.



7 Bitonic Sorter [16]



(8)

suppose for purpose of contradiction that the n/w merges all zero-one sequences but there exists a sequence of arbitrary number that network doesn't correctly merge

There exists input sequences $(a_1, a_2 \ldots a_{\frac{n}{2}})^1$ & $(a_{\frac{n}{2}+1} \ldots a_n)^2$
containing elements a_i from 1 & a_j from 2 s.t $a_i < a_j$ but n/w places a_j before a_i in ~~op~~ output sequence.

$$f(x) = \begin{cases} 0 & \text{if } x \leq a_i \\ 1 & \text{if } x > a_i \end{cases}$$

Since the n/w places a_j before a_i in the output sequence it follows from lemma 27.1 that it places $f(a_j)$ befor $f(a_i)$ in output seq but as $f(a_j) = 1$ & $f(a_i) = 0$

we obtain contradiction.

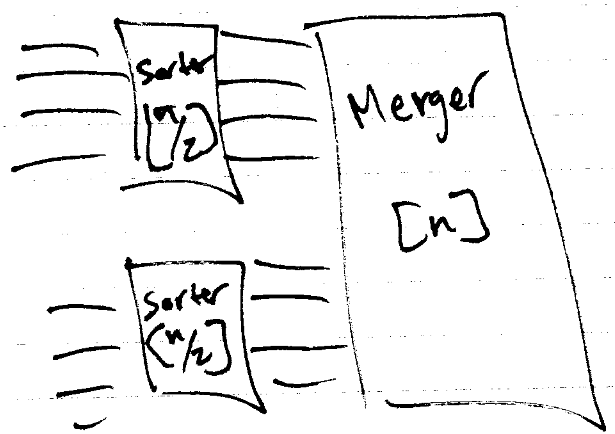
(9) Briefly describe how ~~newtw~~ network Sorter [n] is defined.

Data pass through lg n stages in Sorter [n]. Starting at 1 element sequences sort the sequence using a Merger $[2^k]$

K is the stage from 0 to lg n
Double the sequence length at each subsequent stage.

Each stage has lg n depth and there are lg n stages. Thus, the total depth

$$D[n] = \lg^2 n$$



(10) Length n zero-one bitonic sequences look like $0^i 1^j 0^k$ or $1^i 0^j 1^k$. So the answer is twice # of sequence of form $0^i 1^j 0^k$. This is the # of pairs (i, j) s.t. $0 \le i + j \le n$ where $i, j \ge 1$ together with 0^n. The number of pairs (i, j) s.t. $i + j = m \le n$ $(i, j \ge 1)$ is $m - 1$

$$\sum_{m=1}^{n} (m-1) = \sum_{m=1}^{n-1} m = \frac{n(n-1)}{2}$$

all 0s & all 1s

So number of bitonic sequences is $\frac{n(n-1)}{2} \times 2 + 2$

$$= n(n-1) + 2$$