HW-5

SJSU Students

May 21, 2006

1 Solution to 34.2-8

For any boolean formula ϕ,
ϕ is TAUTOLOGY iff $\neg\ \phi$ is unsatisfiable

Therefore TAUTOLOGY $\leq_p$ $\overline{\text{SAT}}$ —— eq 1

From Theorem 34.9, we have SAT $\in$ NP —— eq 2

Also, Co-NP is defined as the set of langauges L such that $\bar{\text{L}} \in$ NP

Therefore, From eq 1 and eq 2

TAUTOLOGY $\in$ Co-NP

Hence proved

2 Solution to 34.4-6

`Suppose we have a polynomial-time algorithm POLY_ALGO,which decides formula satisfiability.`

`Input of POLY_ALGO is a boolean formula and it outputs 'YES' if the boolean formula is satisfiable. Otherwise,it outputs 'NO'.`

```
Our solution to use POLY_ALGO to find satisfying assignments in polynomial
time is following:

First of all, take given boolean formula as input to POLY_ALGO.

If output is 'NO', we know that there exists no satisfying assignments for
this formula

If output is 'YES', we will do the following:

    for every variable Xi present in boolean formula, where i=1,2,....,n

            assign 1 to Xi, and run POLY_ALGO on the boolean formula with
            remaining unassigned variables

              If POLY_ALGO returns 'YES', keep value of Xi set to 1
              If POLY_ALGO returns 'NO', assign 0 to Xi


At the end we will have input variables having satisfying values.

Total Running Time is O(number of variables * POLY_ALGO).
This is because for every variable, a value is assigned and its
satisfiability is checked.Since POLY_ALGO is polynomial-time algorithm,
the total algorithm is also polynomial-time.
```

3 Solution to 34.5-1

In the given problem we consider subgraph-isomorphism problem which takes 2 graphs G1 and G2 and asks whether G1 is isomorphic to subgraph of G2.

In order to show that subgraph-isomorphism problem is NP-complete, we will first prove that subgraph-isomorphism problem is NP. Then we will prove that it is NP-complete by reducing a known NP-complete problem to it.

First we prove that subgraph-isomorphism problem is NP.

We are given two graphs G1 (having set of vertices V1 and set of edges E1) and G2 (having set of vertices V2 and set of edges E2). Let us also consider a certificate which consists of set V of vertices,a set E of edges and a function t : V1 $\rightarrow$ V. The certificate can be verified in polynomial time by checking the following cases given below:

V $\subseteq$ V2 and $|V| = |V1|$

E $\subseteq$ E2 and $|E| = |E1|$

$\forall$ (u,v) $\in$ E1 : (t(u), t(v)) $\in$ E

t is injective
(where function t is said to be injective if and only if, for every y in the codomain, there is at most one x in the domain such that t(x) = y)

All these checks can be computed in polynomial time. Thus we are able to prove that subgraph-isomorphism problem is NP.

Now we need to prove that subgraph-isomorphism problem is NP-complete.We will prove this by showing that a k-clique problem, which is NP-complete, is polynomial-time reducible to subgraph-isomorphism problem.

Let (G,k) be an instance of clique

Let G2 be a copy of G and let G1 be a complete graph where $|V1| = k$. This can be done in polynomial-time. Then G has a clique of size k iff G2 contains a subgraph isomorphic to G1. Thus

CLIQUE $\leq_p$ subgraph-isomerism

This proves that subgraph-isomorphism problem is NP-complete.

4 Solution to 35.1-1

An example of a graph for which APPROX-VERTEX-COVER always yields a suboptimal solution is a graph with two vertices and a edge between them. This graph has the optimal Vertex Cover consisting of a single vertex. However,APPROX-VERTEX-COVER returns both vertices.