Problem 7.2
Given:

$$\bar{r}_i = (1-u)\bar{p}_i + u\bar{p}_{i+1}, i = 0,1,2 \tag{1}$$

$$\bar{s}_i = (1-u)\bar{r}_i + u\bar{r}_{i+1}, i = 0,1 \tag{2}$$

$$\bar{t}_0 = (1-u)\bar{s}_0 + u\bar{s}_1 \tag{3}$$

Substituting (1) and (2) into (3):

$$\begin{aligned}
\bar{t}_0 &= (1-u)\bar{s}_0 + u\bar{s}_1 \\
&= (1-u)[(1-u)\bar{r}_0 + u\bar{r}_1] + u[(1-u)\bar{r}_1 + u\bar{r}_2] \\
&= (1-u)^2\bar{r}_0 + u(1-u)\bar{r}_1 + u(1-u)\bar{r}_1 + u^2\bar{r}_2 \\
&= (1-u)^2\bar{r}_0 + 2u(1-u)\bar{r}_1 + u^2\bar{r}_2 \\
&= (1-u)^2[(1-u)\bar{p}_0 + u\bar{p}_1] + 2u(1-u)[(1-u)\bar{p}_1 + u\bar{p}_2] + u^2[(1-u)\bar{p}_2 + u\bar{p}_3] \\
&= (1-u)^3\bar{p}_0 + 3u(1-u)^2\bar{p}_1 + 3u^2(1-u)\bar{p}_2 + u^3\bar{p}_3 \\
&= \sum_{i=0}^{3} \binom{3}{i} u^i (1-u)^{3-i}\bar{p}_i = \bar{q}(u)
\end{aligned}$$

Problem 7.3, $\vec{q}\left(\frac{1}{2}\right)$

From Problem 7.1, $\vec{p}_0 = (0, 1)$, $\vec{p}_1 = (1, 2)$, $\vec{p}_2 = (4, 0)$, $\vec{p}_3 = (3, 0)$.

So $\vec{r}_0 = \left(1 - \frac{1}{2}\right)\vec{p}_0 + \frac{1}{2}\vec{p}_1 = \frac{1}{2}(0, 1) + \frac{1}{2}(1, 2) = \left(\frac{1}{2}, \frac{3}{2}\right)$

$\vec{r}_1 = \left(1 - \frac{1}{2}\right)\vec{p}_1 + \frac{1}{2}\vec{p}_2 = \frac{1}{2}(1, 2) + \frac{1}{2}(4, 0) = \left(\frac{5}{2}, 1\right)$

$\vec{r}_2 = \left(1 - \frac{1}{2}\right)\vec{p}_2 + \frac{1}{2}\vec{p}_3 = \frac{1}{2}(4, 0) + \frac{1}{2}(3, 0) = \left(\frac{7}{2}, 0\right)$

Next we calculate the $\vec{s}_i$'s as

$\vec{s}_0 = \left(1 - \frac{1}{2}\right)\vec{r}_0 + \frac{1}{2}\vec{r}_1 = \frac{1}{2}\left(\frac{1}{2}, \frac{3}{2}\right) + \frac{1}{2}\left(\frac{5}{2}, 1\right) = \left(\frac{6}{4}, \frac{5}{4}\right)$

$\vec{s}_1 = \left(1 - \frac{1}{2}\right)\vec{r}_1 + \frac{1}{2}\vec{r}_2 = \frac{1}{2}\left(\frac{5}{2}, 1\right) + \frac{1}{2}\left(\frac{7}{2}, 0\right) = \left(\frac{12}{4}, \frac{2}{4}\right)$

Finally, we calculate the $\vec{t}_0$ as :

$\vec{t}_0 = \left(1 - \frac{1}{2}\right)\vec{s}_0 + \frac{1}{2}\vec{s}_1 = \frac{1}{2}\left(\frac{6}{4}, \frac{5}{4}\right) + \frac{1}{2}\left(\frac{12}{4}, \frac{2}{4}\right) = \left(\frac{18}{8}, \frac{7}{8}\right)$

For q(3/4),

$u = \frac{3}{4}$

$$
\begin{aligned}
\bar{r}_0 &= \frac{1}{4}\bar{p}_0 + \frac{3}{4}\bar{p}_1 \\
\bar{r}_1 &= \frac{1}{4}\bar{p}_1 + \frac{3}{4}\bar{p}_2 \\
\bar{r}_2 &= \frac{1}{4}\bar{p}_2 + \frac{3}{4}\bar{p}_3 \\
\bar{s}_0 &= \frac{1}{4}\bar{r}_0 + \frac{3}{4}\bar{r}_1 \\
&= \frac{1}{4}(\frac{1}{4}\bar{p}_0 + \frac{3}{4}\bar{p}_1) + \frac{3}{4}(\frac{1}{4}\bar{p}_1 + \frac{3}{4}\bar{p}_2) \\
&= \frac{1}{16}\bar{p}_0 + \frac{6}{16}\bar{p}_1 + \frac{9}{16}\bar{p}_2 \\
\bar{s}_1 &= \frac{1}{4}\bar{r}_1 + \frac{3}{4}\bar{r}_2 \\
&= \frac{1}{4}(\frac{1}{4}\bar{p}_1 + \frac{3}{4}\bar{p}_2) + \frac{3}{4}(\frac{1}{4}\bar{p}_2 + \frac{3}{4}\bar{p}_3) \\
&= \frac{1}{16}\bar{p}_1 + \frac{6}{16}\bar{p}_2 + \frac{9}{16}\bar{p}_3 \\
\bar{t}_0 &= \frac{1}{4}\bar{s}_0 + \frac{3}{4}\bar{s}_1 \\
&= \frac{1}{4}(\frac{1}{16}\bar{p}_0 + \frac{6}{16}\bar{p}_1 + \frac{9}{16}\bar{p}_2) + \frac{3}{4}(\frac{1}{16}\bar{p}_1 + \frac{6}{16}\bar{p}_2 + \frac{9}{16}\bar{p}_3) \\
&= \frac{1}{64}\bar{p}_0 + \frac{9}{64}\bar{p}_1 + \frac{27}{64}\bar{p}_2 + \frac{27}{64}\bar{p}_3 \\
&= \frac{1}{64}(0,1) + \frac{9}{64}(1,2) + \frac{27}{64}(4,0) + \frac{27}{64}(3,0) \\
&= (0, \frac{1}{64}) + (\frac{9}{64}, \frac{18}{64}) + (\frac{108}{64}, 0) + (\frac{81}{64}, 0) \\
&= (\frac{198}{64}, \frac{19}{64})
\end{aligned}
$$

Problem 7.6

$$\begin{aligned}
q(u) &= \bar{r}_0 H_0(u) + \bar{r}_1 H_1(u) + \bar{r}_2 H_2(u) + \bar{r}_3 H_3(u) \\
&= (0,1)H_0(u) + (3,3)H_1(u) + (-3,0)H_2(u) + (3,0)H_3(u) \\
&= (0,1)(2u^3 - 3u^2 + 1) + (3,3)(u^3 - 2u^2 + u) + (-3,0)(u^3 - u^2) + (3,0)(-2u^3 + 3u^2)
\end{aligned}$$