Homework 2

SJSU Students

March 17, 2006

1.6 Give state diagrams of DFA's recognizing the following languages. In all parts the alphabet is {0,1}

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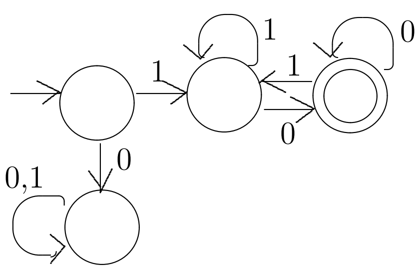
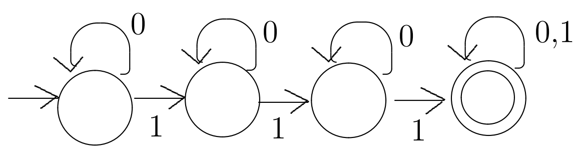
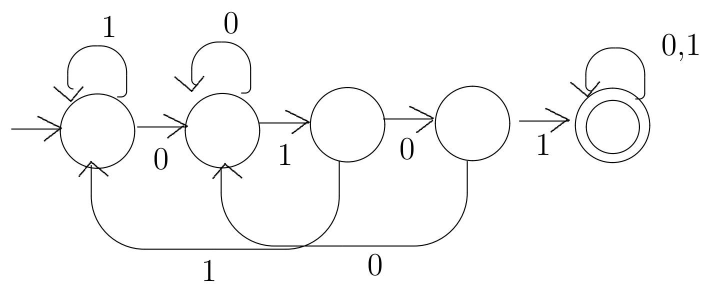
(a) $\{w \mid w$ **begins with a 1 and ends with a 0**$\}$



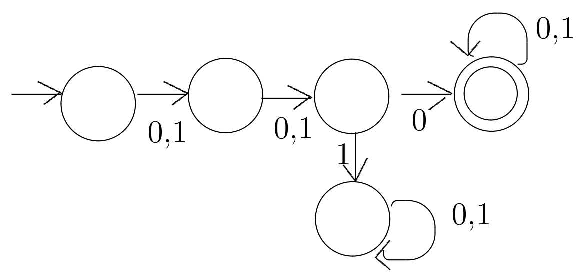
(b) $\{w \mid w$ **contains at least three** 1**'s** $\}$



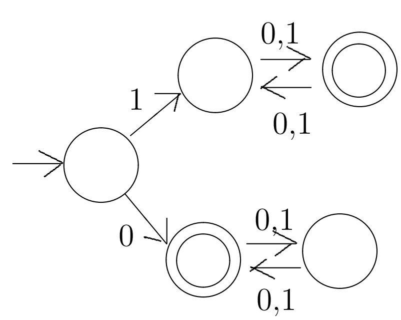
(c) $\{w \mid w$ **contains the substring** 0101 $\}$



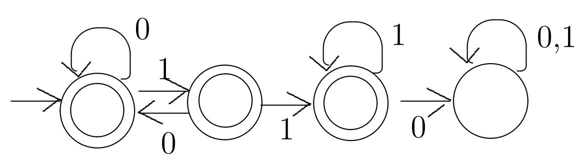
(d) $\{w \mid w$ **length at least** 3 **and its third symbol is a** 0 $\}$



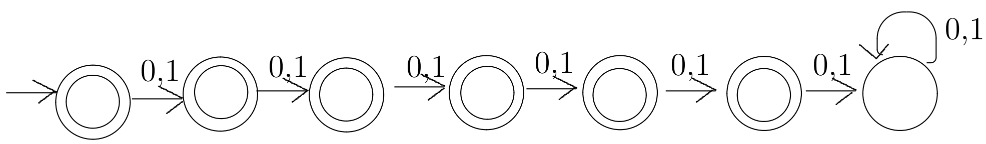
(e) $\{w \mid w$ **starts with an** 0 **and has odd length, or starts with a** 1 **and has even length** $\}$



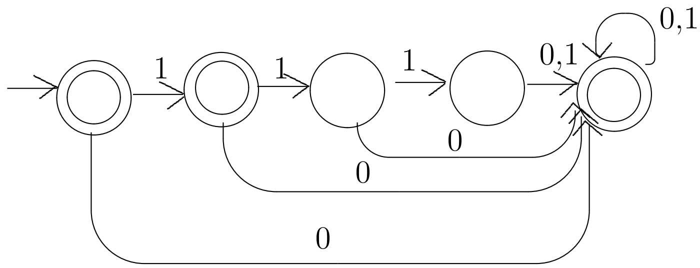
(f) $\{w \mid w$ **doesn't contain the substring** 110 $\}$



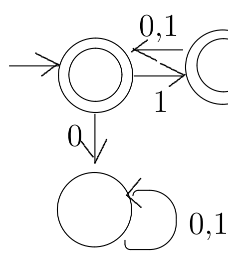
(g) $\{w \mid$ **the length of** w **is at most** 5 $\}$



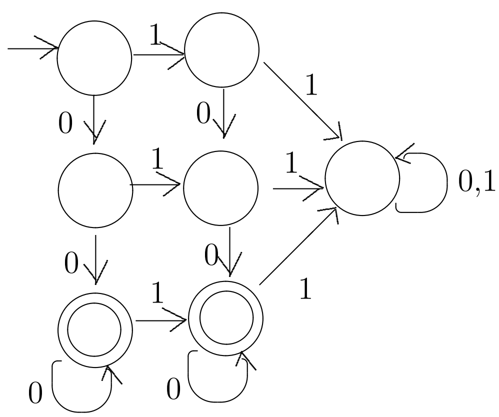
(h) $\{w \mid w$ **is any string except** 11 **and** 111 $\}$



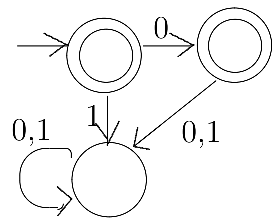
(i) $\{w \mid$ **every odd position of** w **is a** 1 $\}$



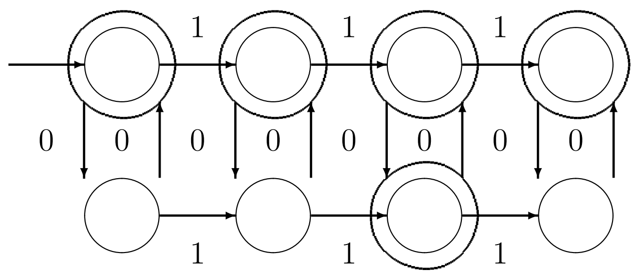
(j) $\{w \mid w$ **contains at least two** 0**s and at most one** 1 $\}$



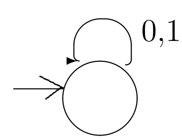
(k) **{ϵ, 0 }**



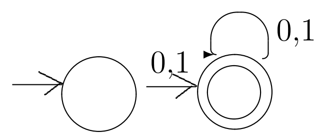
(l) **{w | w contains an even number of 0s, or contains exactly two 1s }**



(m) **The empty set**



(n) All strings except the empty string



1.10 **Use the construction given in the proof of Theorem 1.49 to give the state diagrams of NFAs recognizing the star of the language described in**

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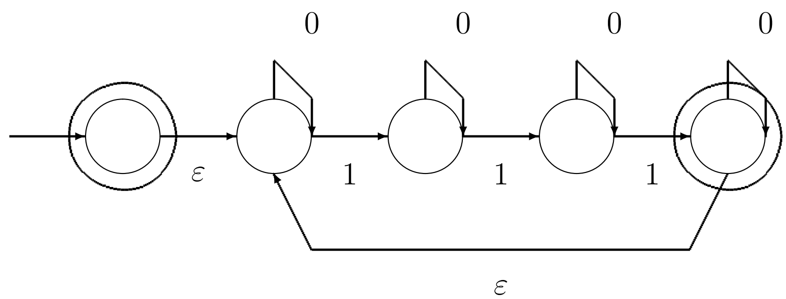
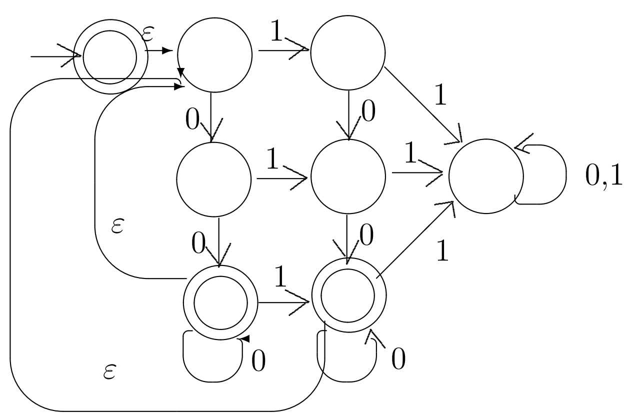
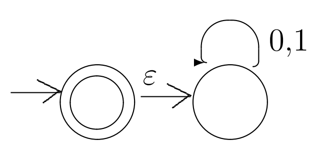
(a) **Exercises 1.6b.**



(b) **Exercise 1.6j.**



(c) **Exercise 1.6m**



1.22 In certain programming languages, comments appear between delimiters such as /# and #/ . Let C be the language of all valid delimited comment strings. A member of C must begin with /# and end with #/ but have no intervening #/. For simplicty, we'll say that the comments themselves are written with only the symbols a and b; hence the alphabet of C is $\Sigma = \{a, b, /, \#\}$

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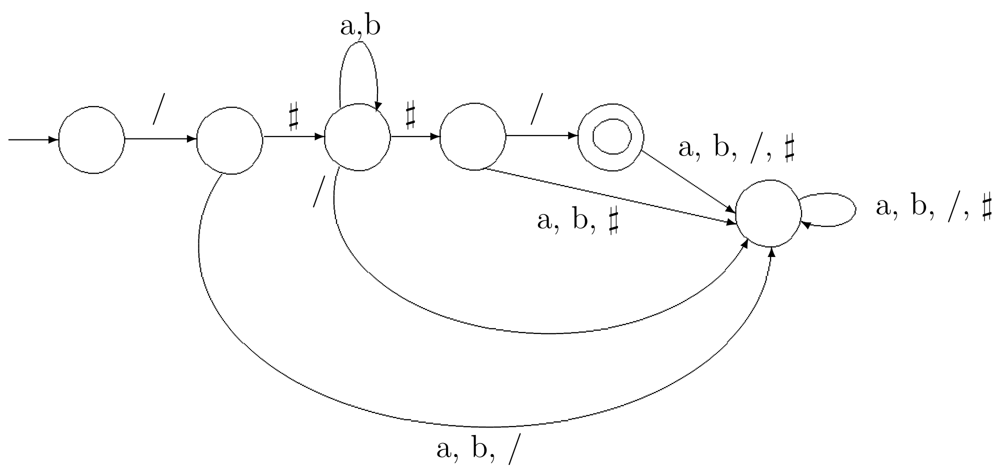
(a) Give a DFA that recognizes C.



(b) Give a regular expression that generates C.

$$/\sharp(\mathrm{a} \bigcup \mathrm{b})^*\sharp/$$

1.24 Give the sequence of the states entered and the output produced in each of the following parts.

(a) T_1 **on input** 011

States: q1,q1,q1,q1 output: 000

(b) T_1 **on input** 211

States: q1,q2,q2,q2 output: 111

(c) T_1 **on input** 121

States: q1,q1,q2,q2 output: 011

(d) T_1 **on input** 0202

States: q1,q1,q2,q1,q2 output: 0101

(e) T_2 **on input** b

States: q1,q3 output: 1

(f) T_2 **on input** $bbab$

States: q1,q3,q2,q3,q2 output: 1111

(g) T_2 **on input** $bbbbbb$

States: q1,q3,q2,q1,q3,q2,q1 output: 110110

(h) T_2 **on input** ε

States: q1 output: ε

1 1.64 Let N be an NFA with k states that recognizes some language A.

a. Show that, if A is nonempty. A contains some string of length at most k. Due to the pigeon principle if there are only k states and we know that some language A is recognized by N, then the length of the string A can not be any longer than k in order to be recognized by N.
b. Show that, by giving an example, that part (a) is not necessarily true if you replace both A's by $\overline{A}$.
c. Show that, if $\overline{A}$ is nonempty, $\overline{A}$ contains some string of length at most 2^k.
d. Show that the bound given in part (c) is nearly tight; that is, for each k,

demonstrate an NFA recognizing a language A_k where $\overline{A_k}$ is nonempty and where $\overline{A_k}$'s shortest member strings are of length exponential in k. Come as close to the bound in (c) as you can.

1.1 For each of the following languages, give the minimum pumping length and justify your answer.

(a) 0001^*

Min. pumping length is 4. x= 000, y = first 1, and z = the rest.

(b) 0^*1^*

Min. pumping length = 1. x = ε, y = first character,and z = the rest.

(c) $001 \cup 0^*1^*$

Using the second part the min. pumping length = 1. x = ε, y = first character,and z = the rest. The first part is a specific case of the second part of the union.

(d) $0^*1^+0^+1^* \bigcup \mathbf{10^*1}$

Min. pumping length is 3. Either x = ε, y = first character of s, and z is the rest, or x = 1, and y = 0, and z = the rest.

(e) $(01)^*$

Min. pumping length = 1. There are no strings in the language of length 1, so every string in the language of length 1 can be pumped. For length ≥ 2, we can take x = ε, y = 01,and z = the rest.

(f) ε

Min. pumping length = 1. The empty string can't be pumped. As there are no strings in the language of length greater than 0, every string in the language of length > 0 can be pumped.

(g) $1^*01^*01^*$

Min. pumping length = 3 . Need to see both 0s before know have a string in the language.

(h) $10(11^*0)^*0$

Min. pumping length = 4. x = 10, y = 11 ,and z = the rest.

(i) 1011

Min. pumping length = 5. As there are no strings in the language of length greater than or equal to 5, all of the strings in the language of length greater than 5 can be pumped.

(j) Σ^*

Min. pumping length = 1. x = ε, y = any character in Σ ,and z = the rest.

1.64 Let N be an NFA with k states that recognizes some language A.

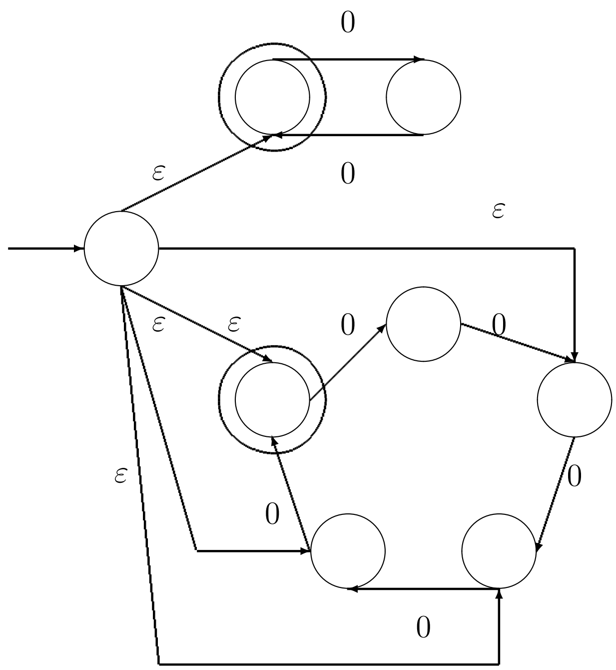
(a) Show that, if A is nonempty. A contains some string of length at most k.

As A is nonempty, one of N's states must be an accept state and there must be some path from the start state to the accept state. Let P be the shortest such path. As we could obtain a shorter path by deleting cycles if P had cycles, we know P does not have cycles. So starting at the start state, each edge traversed in P yields a state not seen so far on the path. As there are only k states P must be of length at most k. Concatenate the symbols on the edges of this path. This will be a string accepted by N of length at most k.

(b) Show by giving an example, that part (a) is not necessarily true if you replace both A's by $\bar{A}$.

That is, we need to show that there is an $\bar{A}$, such that A is recognized by a k state machine, but the first string in $\bar{A}$ has length greater than k.

Consider the machine below over the unary alphabet $\{0\}$. It has 8 states and recognizes the union of the two language $L_1 = \{w \mid |w| \equiv 0 \bmod 2\}$ and $L_2 = \{w \mid |w| \not\equiv 4 \bmod 5\}$. So w will be in $\bar{A}$ if w has length congruent to both $1 \bmod 2$ and $4 \bmod 5$. The first such length is 9 giving the result.



(c) Show that, if $\bar{A}$ is nonempty, $\bar{A}$ contains some string of length at most 2^k.

By using the powerset construction, we can convert N into a DFA with at most 2^k states recognizing A. Let D be this machine and let F be its final state and Q its set of state. Let $\bar{D}$ be the same machine as D except we set its final states to be $Q - F$. Then $\bar{D}$ will recognize $\bar{A}$ and $\bar{D}$ has 2^k. Any DFA can be trivially made into an NFA so we can apply part (a) to the machine $\bar{D}$ to get the result.

(d) Show that the bound given in part (c) is nearly tight; that is, for each k, demonstrate an NFA recognizing a language A_k where $\bar{A}_k$ is nonempty and where $\bar{A}_k$'s shortest member strings are of length exponential in k. Come as close to the bound in (c) as you can.

In class, this was made into a two point bonus question where the hint was to generalize the idea of part (b).