CS154 Homework 1

SJSU Students

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0.2

a. $\{ 1, 10, 100 \}$
b. $\{ x | x \in \boldsymbol{Z} \text{ and } x > 5 \}$
c. $\{ 0, 1, 2, 3, 4 \}$
d. $\{ aba \}$
e. $\{ \epsilon \}$
f. $\emptyset$

0.3

a. No, A is not a subset of B since z $\notin B$ and z $\in A$.
b. Yes, $B \subseteq A$ since each element of B is in A.
c. $A \cup B = A = \{ x, y, z \}$
d. $A \cap B = B = \{ x, y \}$
e. $A \times B = \{(x,x), (x,y), (y,x), (y,y), (z,x), (z,y)\}$
f. $\wp(B) = \{\emptyset, \{x\}, \{y\}, \{x,y\}\}$

0.6

a. $f(2) = 7$
b. The range of $f = \{6, 7\}$ / The domain of $f = \boldsymbol{X} = \{1, 2, 3, 4, 5\}$
c. $g(2, 10) = 6$
d. The range of $g = \boldsymbol{Y} = \{6, 7, 8, 9, 10\}$ The domain of $g = \boldsymbol{X} \times \boldsymbol{Y}$ where $\boldsymbol{X} \times \boldsymbol{Y} = \{(1,6), (1,7), (1,8), (1,9), (1,10), (2,6), (2,7), (2,8), (2,9), (2,10), (3,6), (3,7), (3,8), (3,9), (3,10), (4,6), (4,7), (4,8), (4,9), (4,10), (5,6), (5,7), (5,8), (5,9), (5,10)\}$
e. $g(4, f(4)) = g(4, 7) = 8$

0.7

a. Reflexive and symmetric but not transitive
$\boldsymbol{R} = \{ (x,y) | x, y \in \boldsymbol{Z} \text{ and } xy \geq 0 \}$
It is reflexive because the relation holds when $x = y$ and the relation contains all integers.
It is symmetric because xy = yx. e.g. $(-8, 0) \in \boldsymbol{R}$ and $(0, -8) \in \boldsymbol{R}$.
It is not transitive since both xy and yz have to be 0 or a positive number.
e.g. Suppose $(x,y) = (-7,0) \in \boldsymbol{R}$ and $(y,z) = (0,11) \in \boldsymbol{R}$ but $(x,z) = (-7,11) \notin \boldsymbol{R}$ because $xz < 0$.

0.7

b. Reflexive and transitive but not symmetric

$\boldsymbol{R} = \{ (x,y) | x, y \in \boldsymbol{Z} \texttt{ and } x \leq y \}$

It is reflexive because x and y can be equal and so the relation contains all integers.

It is not symmetric because x cannot be greater than y.

e.g. $(-1,5) \in \boldsymbol{R}$, but $(5,-1) \notin \boldsymbol{R}$.

It is transitive since x $\leq$ y and y $\leq$ z. Hence x $\leq$ z.

e.g. Suppose $(x,y) = (-4,0) \in \boldsymbol{R}$ and $(y,z) = (0,0) \in \boldsymbol{R}$.

Then, $(x,z) = (-4,0) \in \boldsymbol{R}$.

c. Symmetric and transitive but not reflexive

$\boldsymbol{R} = \{ (x,y) | x, y \in \boldsymbol{Z} \texttt{ and } xy \geq 1 \}$

It is not reflexive because (0, 0) is not included.

It is symmetric because xy = yx. e.g. $(-2,-5) \in \boldsymbol{R}$ and $(-5,-2) \in \boldsymbol{R}$.

It is transitive since this relation holds as long as both x and y are either positives or negatives so are y and z. e.g. Suppose $(x,y) = (3,6) \in \boldsymbol{R}$ and $(y,z) = (6,12) \in \boldsymbol{R}$. Then, $(x,z) = (3,12) \in \boldsymbol{R}$ because both x and z are positives. This also applys when x, y, and z are negatives.

0.10

The error in the following proof that 2 = 1 is dividing both side by (a - b) in the **Step5** because (a - b) = 0. Division of both sides of an equation by the same quantity is valid as long as this quantity is not zero.

Step1. Let $a = b$

Step2. Multiply both side by a to obtain a^2 = ab

Step3. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$

Step4. Factor each side $(a+b)(a-b) = b(a-b)$

Step5. Divide each side by $(a-b)$ to get $a+b=b$

Step6. Let $a = b = 1$

Step7. 2 = 1

0.12

Every graph with 2 or more nodes contains two nodes that have equal degrees.

Proof: Let n be the number of nodes of a graph where n $\geq$ 2. Let deg(v_i) be the degree, the number of edges at each node, v_1, v_2, ..., v_n. An isolated node cannot be present when n = 2 by definition of a graph.

Then, $1 \leq$ deg(v_i) $\leq n-1$.

By applying the Pigeonhole Principle, there will be $n-1$ pigeonholes (possible number of edges at each node) and n pigions (the number of nodes of a graph). Hence, there must be at least two nodes that have equal degrees .