(1) $CFG = (V, \Sigma, R, A)$

$V = \{A\}$

$\Sigma = \{(, )\}$

$R = \{ A \to (A),$

$A \to AA,$

$A \to \lambda \}$

Group #2

Q4 To prove that CFG on $\{(, )\}$ is not regular:

$L = \{w \mid w$ has an equal no. of $($'s & $)$'s$\}$

Proof: Suppose M is a DFA that regcognise L

Let $p$ be M's pumping length.
consider the string $w = (^p )^p$. This
String is in the language & has length $> p$
— so by pumping lemma
$w = xyz$, where $|xy| \le p$, $|y| > 0$
& where $xy^iz$ is in the language for all
$i \ge 0$. That means $x = (^k$ & $y = )^j$ where
$k + j \le p$ & $j > 0$.
But then taking $i = 0$, $xz = (^{p-j} )^p$
Should be in L
As $p - j$ is less than / or not equal to $p$
there will be one less ( on the right
hand side of the ~~CFG~~ L. Eg: ((A)))
~~Thus the langu~~ ~~B~~ which is not
present in the ~~CFG~~ L ~~$\Rightarrow$~~ $\Leftarrow$ Thus not
regular.

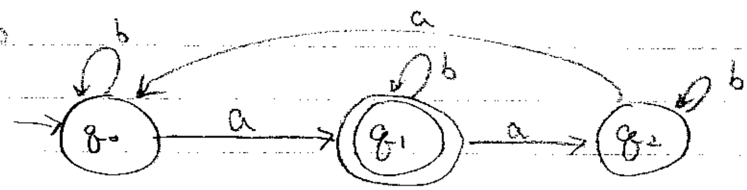
#3 Construct DFA

a) $L_1 = \{w \mid n_a(w) = 1 \bmod 3\}$

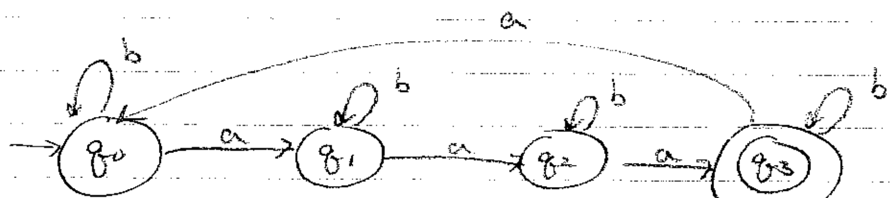
b) $L_2 = \{w \mid n_a(w) = 3 \bmod 4\}$

c) $L_3 = \{w \mid n_a(w) = 3 \bmod 12\}$

a)

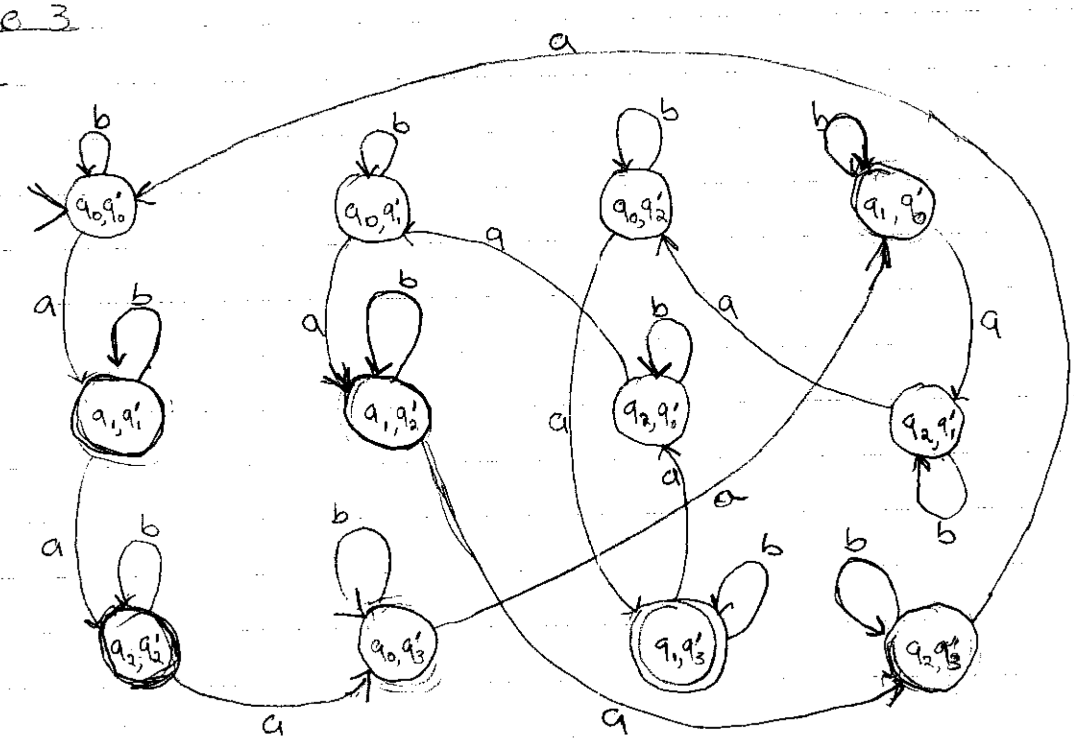



b)



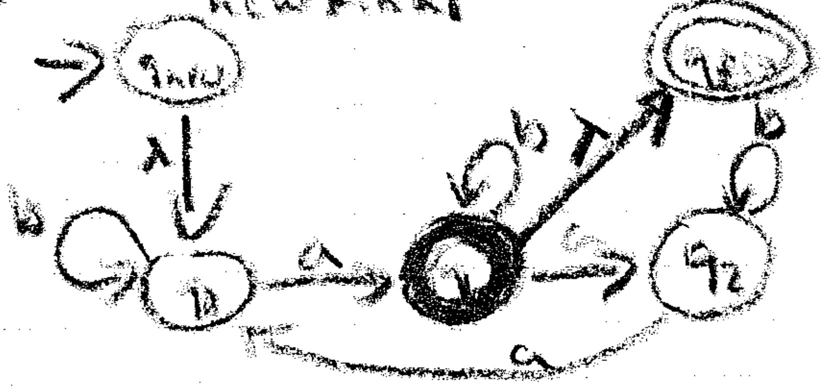
Exercise 3

Part C



4

(a) Add new start + transitions
new final

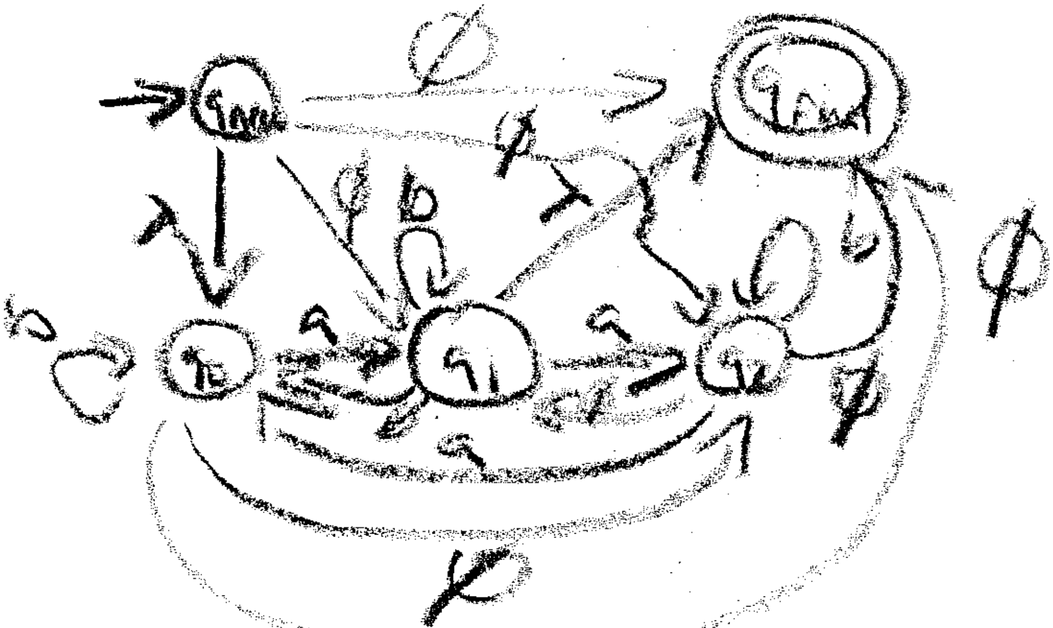


(b) No state had multiple arrows

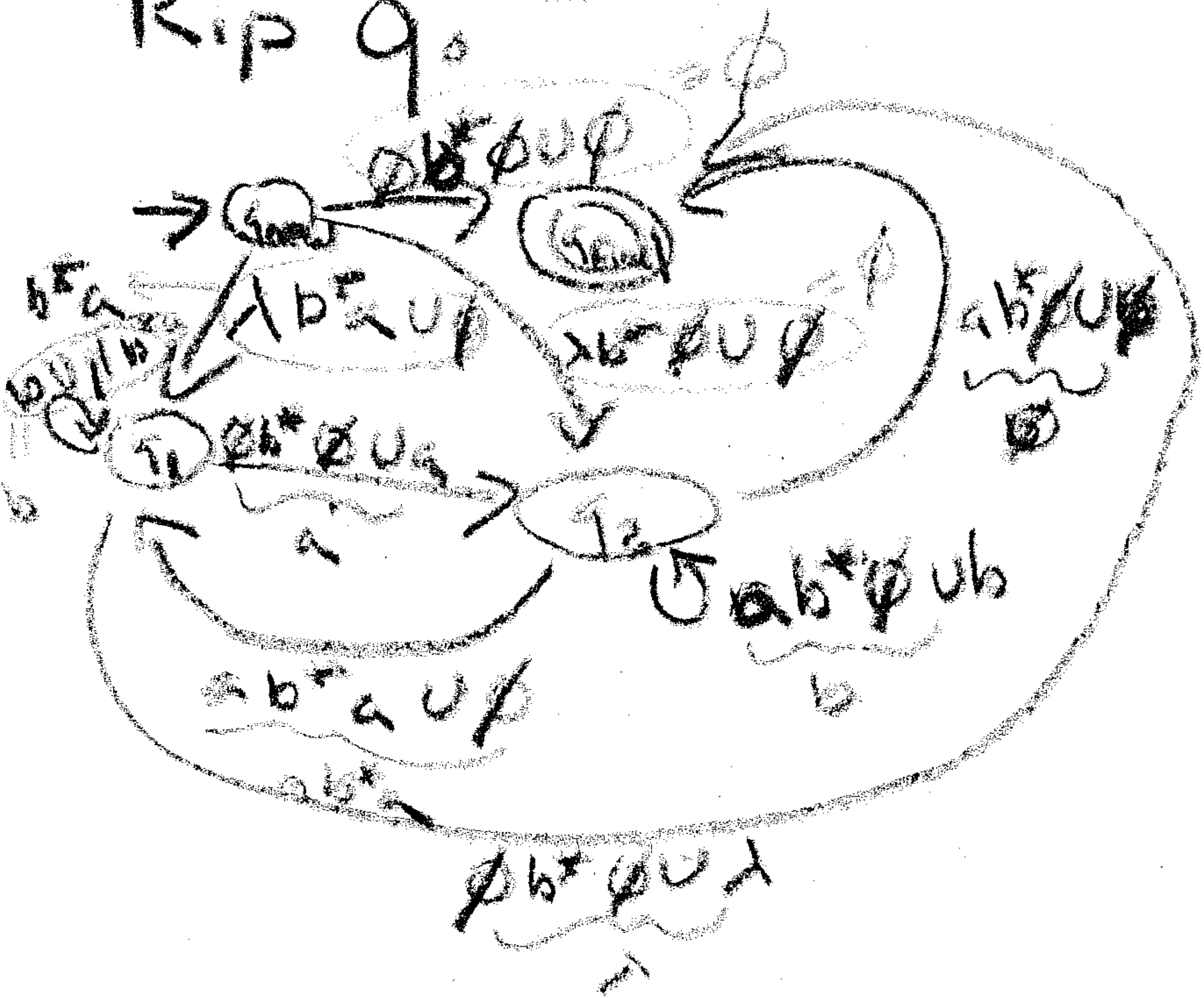
(c) Add $\emptyset$ transitions

See next page $\Rightarrow$

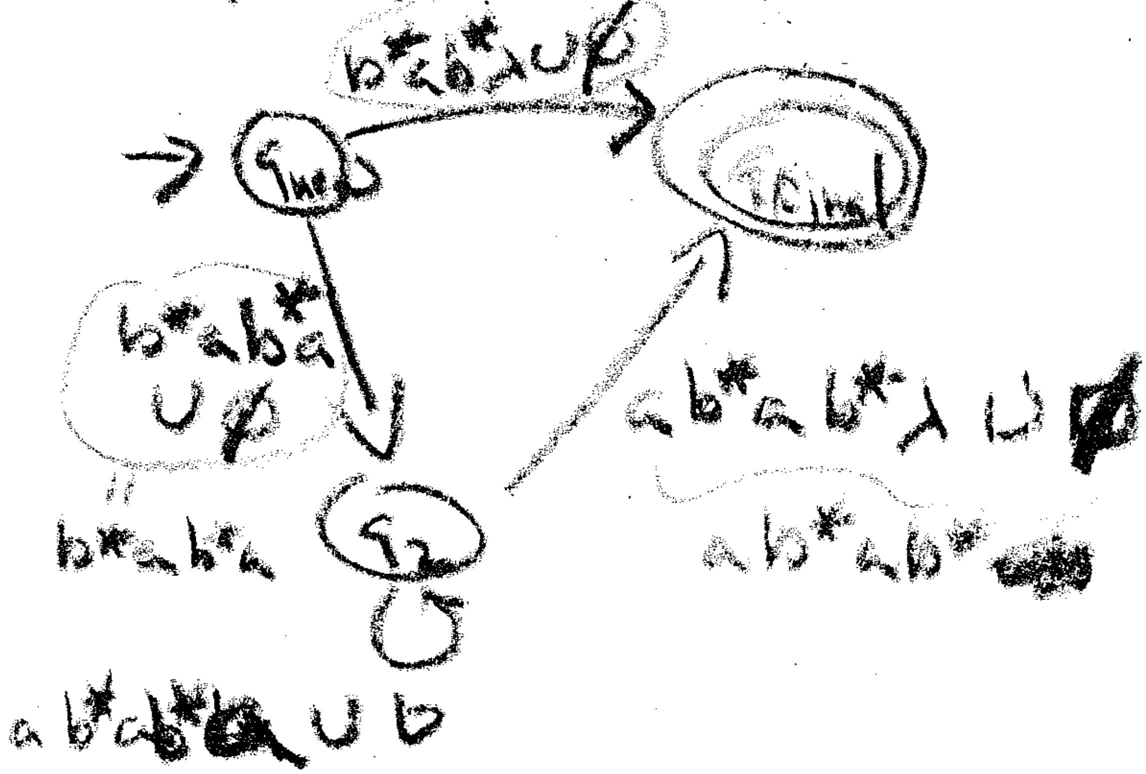
C cont'd



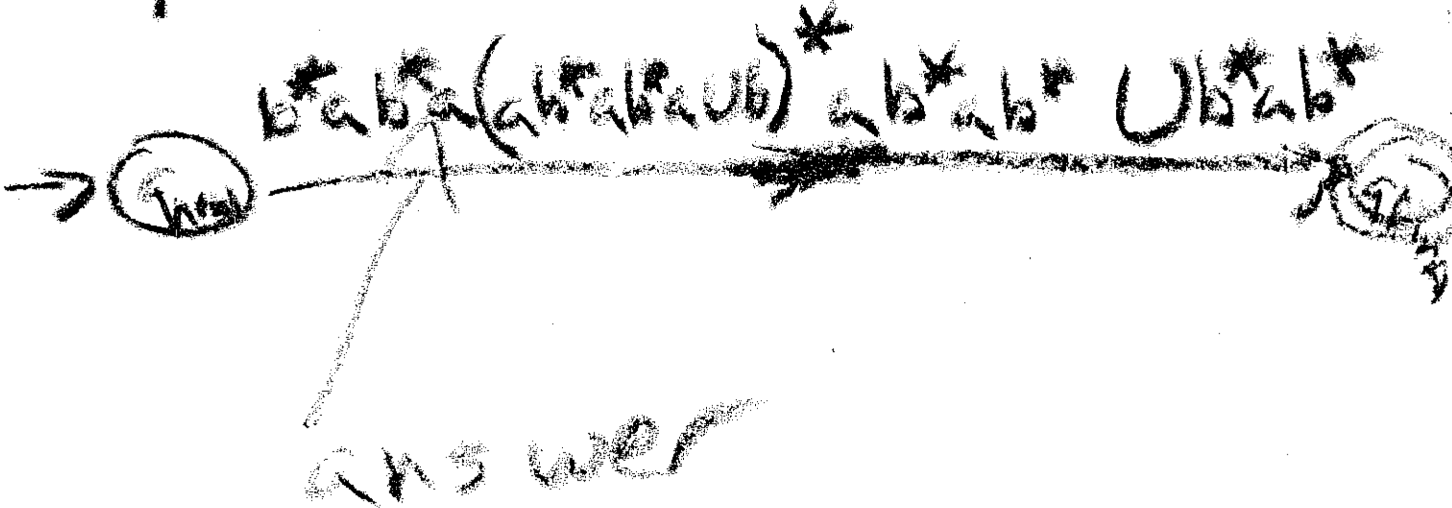
Rip $q_0$



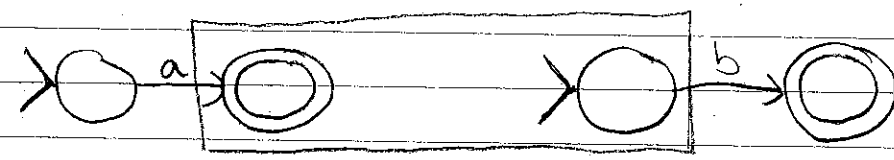
# Rip $q_1$

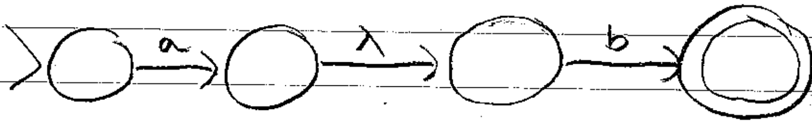

b*ab*
b*ab*λ ∪ ∅
q_new
q_final
b*ab*a ∪ ∅
= b*ab*a
q_2
ab*ab*λ ∪ ∅
ab*ab*
ab*ab*a ∪ b

# Rip $q_2$

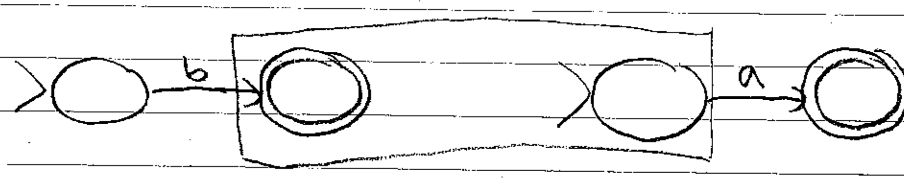

b*ab*a(ab*ab*a ∪ b)* ab*ab* ∪ b*ab*
q_new
q_final
answer

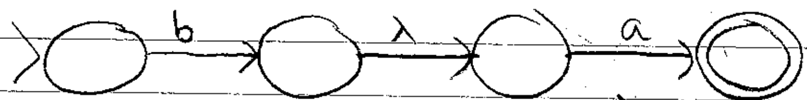
5



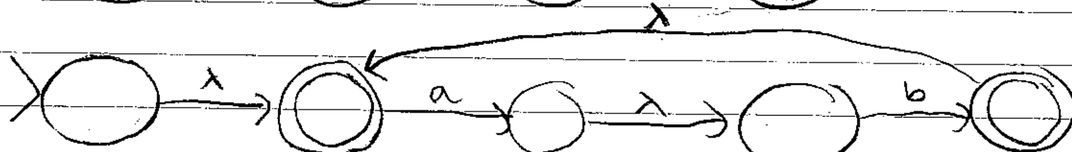


ab

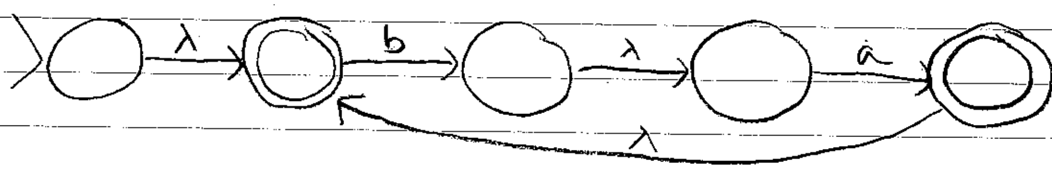




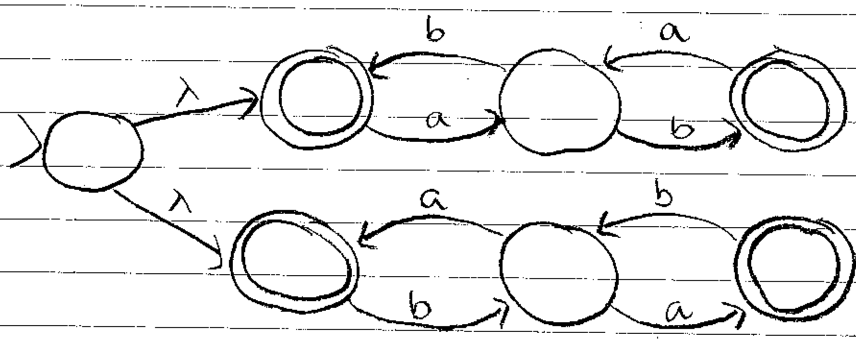
ba



$(ab)^*$



$(ba)^*$



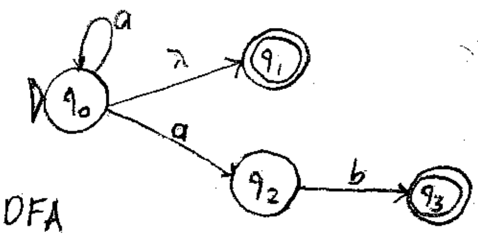
$(ab)^* \cup (ba)^*$

10 $S_1$ = start variable for $nfa_1$
$S_2$ = start variable for $nfa_2$

to union $nfa_1 \cup nfa_2$, we add a new start variable $S$

$\Rightarrow$ $S \rightarrow S_1$
$S \rightarrow S_2$

Q6 Consider NFA




Show step by step conversion to equivalent DFA

Ans NFA - to - DFA

1. Create a transition graph $G_D$ with start state vertex $\{q_0, q_1\}$. ($q_1$ is part of start state due to the $\lambda$-transition, empty string going to $q_1$)
See Fig 1.


Fig 1:

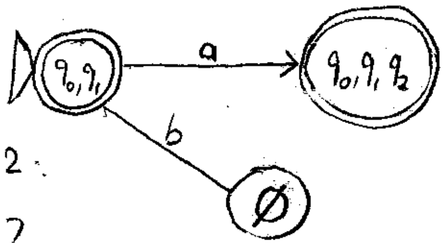



Fig 2:

2. Where does a & b go from $\{q_0, q_1\}$?
Note: $\delta(\{q_0, q_1\}, a) = \{q_0, q_1 q_2\}$ Similarly, $\delta(\{q_0, q_1\}, b) = \emptyset$
Draw the transition a to state $\{q_0, q_1 q_2\}$ and b to the empty state $\emptyset$
Fig 2:

3. Repeat step 2 for any more states in the NFA. Fig 3

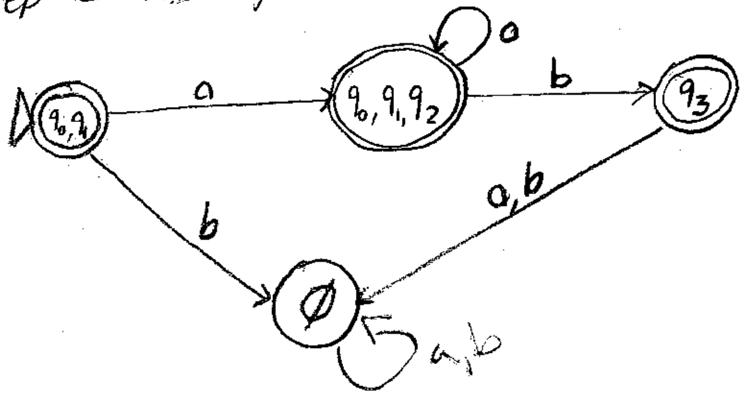



Fig 3:

4. Any states which have an element of the final states of the NFA are now final states.

Q7 Do state minimization on the DFA in Q6.

| | $\{\}$ | $\{q_0, q_1\}$ | $\{q_0, q_1 q_2\}$ | $\{q_3\}$ |
|---|---|---|---|---|
| $\{\}$ | | X | X | X |
| $\{q_0, q_1\}$ | X | | X | X |
| $\{q_0, q_1 q_2\}$ | X | X | | X |
| $\{q_3\}$ | X | X | X | |

X: distinguishable.

Ans:
It cannot be reduced any further, thus it is already in minimized form.

(8) If A & B are regular languages, the A/B is also Regular.

let $M(Q, \Sigma, \partial, q_0, F)$ be a DFA for A.

let $M'(Q, \Sigma, \partial, q_0, F)$ be a DFA for B.

So, a string v is in $A/B = \frac{L(M)}{L(M')}$ if $\partial^*(q_0, v) = q_i$

~~from~~ for some i, $\partial^*(q_i, w) \in F$, & $w \in L(M')$.

Let $M_i = (Q, \Sigma, \partial, q_i, F)$, we know that $L(M_i) \cap L(M')$ is regular,

and $L(M_i) \cap L(M')$ is non-empty, we can check this by

seeing if a final state is reachable by initial state.

So, we can make machine for A/B as $(Q, \Sigma, \partial, q_0, F')$, where

$F'$ are states in Q such that $L(M_i) \cap L(M')$ is non-empty.

(9) $h : \Sigma \to \Gamma^*$

$h: \{0^n 1^m 2^{n-m} \mid n \geq m \geq 0\} \to \{0^n 1^n \mid n \geq 0\}$

$h(w) = \quad h: 0 \to 0$

$\quad h: 1 \to 1$

$\quad h: 2 \to 1$

$h(0^n 1^m 2^{n-m}) = 0^n 1^m 1^{n-m} = 0^n 1^n$

Since we know $0^n 1^n$ is not regular, and $0^n 1^m$ is a hom. image of $\{0^n 1^m 2^{n-m} \mid n \geq m \geq 0\}$, then $\{0^n 1^m 2^{n-m} \mid n \geq m \geq 0\}$ is also not regular and therefore not closed under homomorphism.