Conversions and Minimization

CS154

Chris Pollett

Feb 12, 2007.

Outline

- NFA to DFA Conversion
- State Minimization

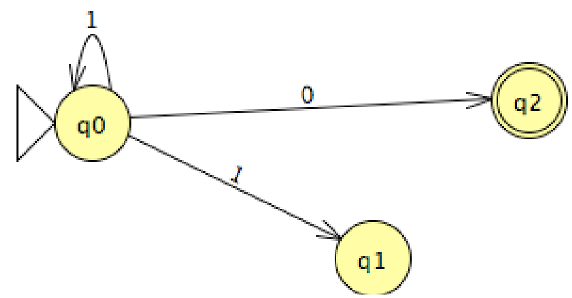
Equivalence of NFAs and DFAs

Theorem Any language recognized by an NFA is recognized by some DFA.

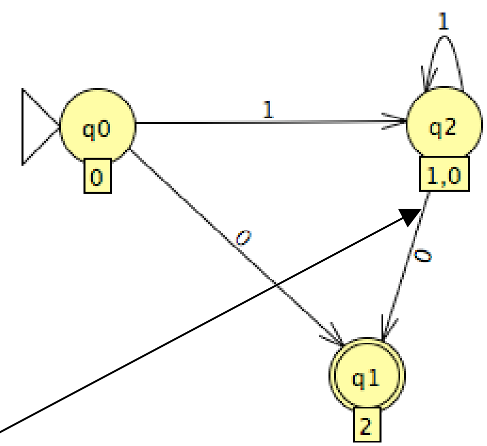
Proof: Given an NFA N= $(Q, \Sigma, \delta, q, F)$ we want to simulate how it acts on a string w with a DFA, M= $(Q', \Sigma, \delta', q', F')$. The idea is we want to keep track of what possible states it could be in after reading the first m characters of w. Let $Q' = P(Q)$. The alphabet is the same. For each $R \in Q'$ and $a \in \Sigma$, let $\delta'(R,a) = \{q \in Q \mid q \in E(\delta(r,a))$ for some $r \in R\}$. Here $E(q')$ is the set of states reachable from q' following only ε transitions. Let $q' = E(q)$. Let $F' = \{R \in Q' \mid R$ contains an accept state of N$\}$.

Example Conversion

- Here is an initial NFA



- Here is the result of the conversion, where unreachable states have been removed
- JFLAP let’s you step through the conversion process



- The subscript beneath a state after the conversion corresponds to a set of states from the original NFA

State Minimization

- We say two states p,q of a DFA M are **indistinguishable** if $\delta^*(p,w) \in F$ implies $\delta^*(q,w) \in F$ and $\delta^*(p,w) \notin F$ implies $\delta^*(q,w) \notin F$.
- Otherwise, p,q are said to be **distinguishable.**
- Let $p \sim_I q$, if p and q are indistinguishable. Notice this is an equivalence relation.
- We now present an algorithm to find the minimal DFA equivalent to M.
- The idea is to first compute the equivalence classes of the indistinguishable equivalence relation. Then make one state for each equivalence class, and make an appropriate new transition function.

Procedure for Equivalence Classes

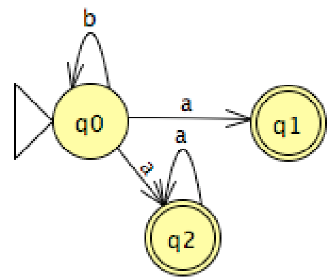
1. Remove all inaccessible states. This can be done by check for each state if there is a simple path from the start state to it.
2. Consider all pairs (p,q). If $p \in F$ but $q \notin F$ or vice versa, then mark the pair (p,q) distinguishable.
3. Repeat until no previously unmarked pairs are marked:
   a) For all pairs (p,q) and all $a \in \Sigma$, compute $\delta(p,a) = p_a$ and $\delta(q,a) = q_a$. If the pair (p_a, q_a) is marked as distinguishable, mark (p,q) as distinguishable.

Procedure to Build Minimal Automaton

1. Use procedure of last slide to generate state equivalence classes for original automata.
2. For each equivalence class $[p] = \{q \mid p \sim_I q\}$ create a new state.
3. For each transition rule $\delta(r,a)=s$ of the original machine, add a transition $\delta([r],a)=[s]$.
4. The initial state of the new machine is $[q_0]$ where q_0 was the state of the machine we are trying to minimize.
5. The final states of the new machine is the set $\{[f] \mid f \in F\}$.

Example

- Consider the NFA:



- In class will go over example