The Pumping Lemma and Closure Properties of CFLs

CS154

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Outline

- Proof of the Pumping Lemma
- Class example

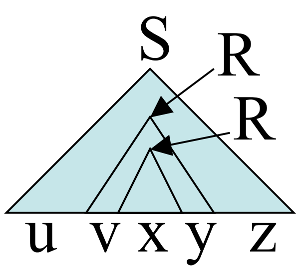
The Pumping Lemma Stated Again

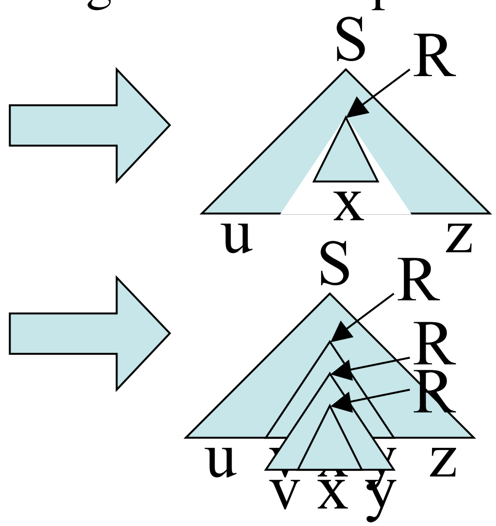
Pumping Lemma for Context Free Languages: If A is a context free language, then there is a number p (the pumping length) where, if s is any string A of length at least p, then s maybe divided into five pieces s= uvxyz satisfying the conditions:

1. for each i>=0, uv^ixy^iz is in A.
2. $|vy| > 0$, and
3. $|vxy| <= p$.

Proof of the Pumping Lemma for CFGs.

Let G be a CFG for our context free language A. Let |V| be the number of variables in G. Let b be the maximum number of symbols on the right hand side of a rule. So the maximum number of leaves a parse tree of height d can have is b^d. We set the pumping length to $p = b^{|V|}+1$. So if s is in A of length bigger than p, its smallest parse tree must be of height greater than |V|+1. So some variable R must be repeated. So we can do the following kind of surgeries on the parse tree to show condition 1 of the pumping lemma:





Condition 2 of the pumping lemma will hold since if v and y were the empty string then the pumped down tree would be a smaller derivations of s contradicting our choice of parse tree. Condition 3 can be guarenteed by choosing R among the laset |V|+1 of the longest path in the tree.

Example

- Prove the language {www | w is a string over {0,1}} is not context-free. Try this for a few minutes in groups, then we’ll work on it on the board.