Homework assignment #3

Page 144, Prob# 3.

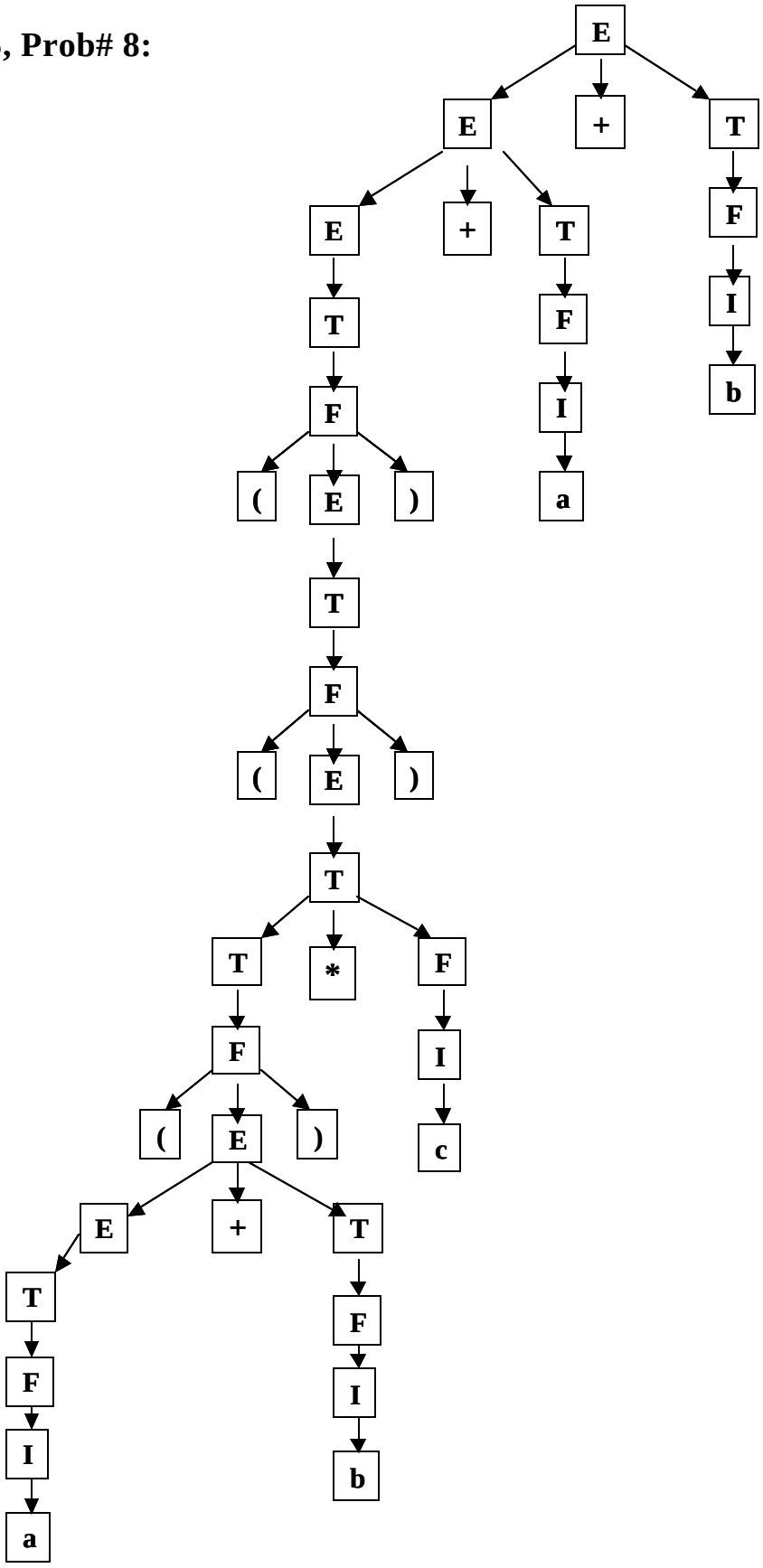
L = $\{a^n b^{n+1}: n >= 2\}$

w = aa(ab)*bbb

So we can find the s-grammar as follows:

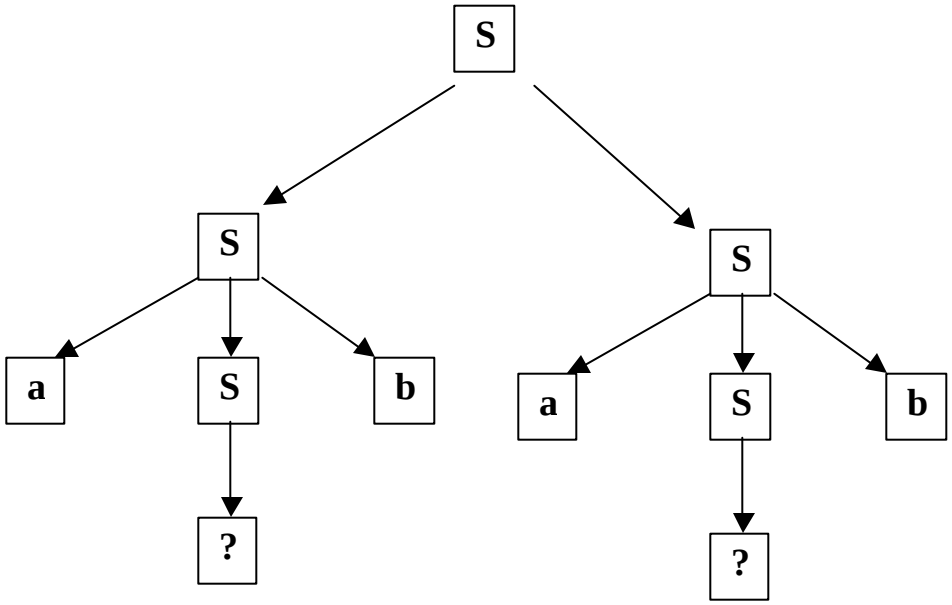
$S \rightarrow aA_1$
$A_1 \rightarrow aA_2$
$A_2 \rightarrow aA_2B_1 \mid bB_2$
$B_1 \rightarrow b$
$B_2 \rightarrow bB_3$
$B_3 \rightarrow b$

Page145, Prob# 8:

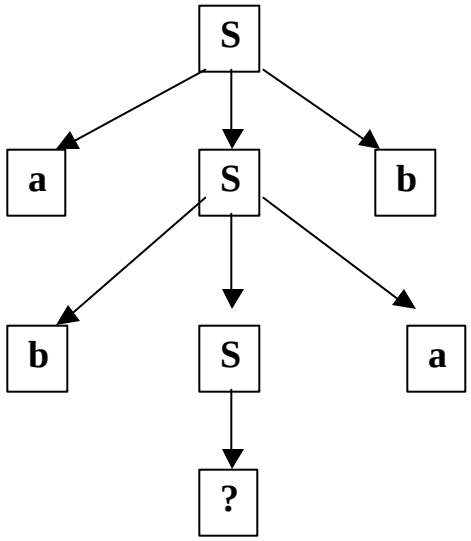


Page 145, Prob#15. The string w = abab has two derivations as shown below. So the grammar is ambiguous.

Derivation 1:



Derivation 2:



Page 145, #16.

Given Grammar with productions:

$S \rightarrow aAB$,
$A \rightarrow bBb$,
$B \rightarrow A|\lambda$
This grammar is actually ambiguous:
S->aAB->abBbB->abbB->abbbBb->abbbb
S->aAB->abBbB->abbBbbB->abbbbB->abbbb

Page 169, Prob# 4.

Given S→ abAB
A→bAB|λ
B→BAa|A|λ

Step 1 :

New start variable to get:

S0 →S, S→abAB, A→bAB|λ, B→BAa|A|λ

Step 2:

1. Remove B→λ
   S0 →S
   S→abA|abAB
   A→bA|bAB|λ
   B→Aa |BAa |A

2. Remove A→λ
   S0 →S
   S→ab|abA|abB|abAB
   A→b|bA|bB|bAB
   B→a|Aa|Ba|BAa|A

Step 3:

Remove unit production

1. Remove B→A
   S0 →S
   S→ab|abA|abB|abAB
   A→b|bA|bB|bAB
   B→ a|Aa|Ba|BAa|b|bA|bB|bAB

2. Remove S0 →S
   S0→ ab|abA|abB|abAB

$S \rightarrow ab|abA|abB|abAB$
$A \rightarrow b|bA|bB|bAB$
$B \rightarrow a|Aa|Ba|BAa|b|bA|bB|bAB$

Step 4: Split –up rule with |RHS| > 2

$S0 \rightarrow ab|aC_1|aC_2|aC_3$, $C_1 \rightarrow bA$, $C_2 \rightarrow bB$, $C_3 \rightarrow bC_4$, $C_4 \rightarrow AB$

$S \rightarrow ab|aD_1|aD_2|aD_3$, $D_1 \rightarrow bA$, $D_2 \rightarrow bB$, $D_3 \rightarrow bD_4$, $D_4 \rightarrow AB$

$A \rightarrow b|bA|bB|bE$, $E \rightarrow AB$

$B \rightarrow a|Aa|Ba|BF_1|b|bA|bB|bF_2$, $F_1 \rightarrow aA$, $F_2 \rightarrow AB$

Step 5 : Put in correct format to obtain Chomsky normal form

$S0 \rightarrow A_1B_1|A_2C_1|A_3C_2|A_4C_3$, $C_1 \rightarrow B_2A$, $C_2 \rightarrow B_3B$, $C_3 \rightarrow B_4C_4$, $C_4 \rightarrow AB$
$A_1 \rightarrow a$, $B_1 \rightarrow b$, $A_2 \rightarrow a$, $A_3 \rightarrow a$, $A_4 \rightarrow a$, $B_2 \rightarrow b$, $B_3 \rightarrow b$, $B_4 \rightarrow b$

$S \rightarrow A_5B_5|A_6D_1|A_7D_2|A_8D_3$, $D_1 \rightarrow B_6A$, $D_2 \rightarrow B_7B$, $D_3 \rightarrow B_8D_4$, $D_4 \rightarrow AB$
$A_5 \rightarrow a$, $B_5 \rightarrow b$, $A_6 \rightarrow a$, $A_7 \rightarrow a$, $A_8 \rightarrow a$, $B_6 \rightarrow b$, $B_7 \rightarrow b$, $B_8 \rightarrow b$

$A \rightarrow b|B_9A|B_{10}B|B_{11}E$, $E \rightarrow AB$, $B_9 \rightarrow b$, $B_{10} \rightarrow b$, $B_{11} \rightarrow b$

$B \rightarrow a|AA_9|BA_{10}|BF_1|b|B_{12}A|B_{13}B|B_{14}F_2$, $F_1 \rightarrow A_{11}A$, $F_2 \rightarrow AB$
$A_9 \rightarrow a$, $A_{10} \rightarrow a$, $B_{12} \rightarrow b$, $B_{13} \rightarrow b$, $B_{14} \rightarrow b$, $A_{11} \rightarrow a$

Page 170 # Prob# 13.

Given the grammar

$S \rightarrow ABb|a$
$A \rightarrow aaA|B$
$B \rightarrow bAb$

Step 1:
Introduce new variable C with rule $C \rightarrow b$ and use it where 'b' is not at 1st position. We get

$S \rightarrow ABC|a$
$C \rightarrow b$
$A \rightarrow aaA|B$
$B \rightarrow bAC$

Step 2:
Introduce new variable D with rule $D \rightarrow a$ and use it where 'a' is not at 1st position. We get

$S \rightarrow ABC|a$
$C \rightarrow b$
$A \rightarrow aDA|\ bAC$
$D \rightarrow a$
$B \rightarrow bAC$

Step 3:
Use $A \rightarrow aDA|\ bAC$ in $S \rightarrow ABC$ to obtain

$S \rightarrow aDABC|bACBC|a$
$A \rightarrow aDA|\ bAC$
$C \rightarrow b$
$D \rightarrow a$
$B \rightarrow bAC$

The grammar is now in Greibach form.

Page 172 Prob#1.

Given, language L with grammar G, where

G: $S \rightarrow AB$

$A \rightarrow BB|a$

$B \rightarrow AB|b$

Part1:

w = aabb

Step1:

$V11 = \{A : A \rightarrow a\} = \{A\}$

$V22 = \{A : A \rightarrow a\} = \{A\}$

$V33 = \{A : A \rightarrow b\} = \{B\}$

$V44 = \{A : A \rightarrow b\} = \{B\}$

Step 2:

$V12 = \{A: A \rightarrow BC, B \in V11 , C \in V22\} = \Phi$

$V23 = \{ A : A \rightarrow BC, B \in V22 , C \in V33\} = \{S,B\}$

$V34 = \{ A: A \rightarrow BC, B \in V33 , C \in V44\} = \{A\}$

Step 3:

$V13 = \{ A: A \rightarrow BC, B \in V11 , C \in V23\}$

$U\{ A: A \rightarrow BC, B \in V12 , C \in V33\} = \{S, B\}$

$V24 = \{ A: A \rightarrow BC, B \in V22 , C \in V34\}$

$U\{ A: A \rightarrow BC, B \in V23 , C \in V44\} = \{A\}$

Step 4:

$V14 = \{ A: A \rightarrow BC, B \in V11 , C \in V24\}$

$U \{ A: A \rightarrow BC, B \in V12 , C \in V34\}$

$U\{ A: A \rightarrow BC, B \in V13 , C \in V44\} = \{A\}$

Since V14 does not have S, therefore, $w \notin L(G)$

Page #172, prob#1, part2.

w = aabba

Step1.

V11 = {A}
V22 = {A}
V33 = {B}
V44 = {B}
V55 = {A}

Step 2:

V12 = {A: A→BC, B∈ V11 , C∈ V22} = Φ
V23 = { A : A→BC, B∈ V22 , C∈ V33} = {S,B}
V34 = { A: A→BC, B∈ V33 , C∈ V44} = {A}
V45 = { A: A→BC, B∈ V44 , C∈ V55} = Φ

Step 3:

V13 = { A: A→BC, B∈ V11 , C∈ V23}
U { A: A→BC, B∈ V12 , ∈ V33}
= {S, B}

V24 = { A: A→BC, B∈ V22 , C∈ V34}
U { A: A→BC, B∈ V23 , C∈ V44}
= {A}

V35 = { A: A→BC, B∈ V33 , C∈ V45}
U { A: A→BC, B∈ V34 , C∈ V55}
= Φ

Step 4:

V14 = { A: A→BC, B∈ V11 , C∈ V24}
U { A: A→BC, B∈ V12 , C∈ V34}
U { A: A→BC, B∈ V13 , C∈ V44}

$= \{A\}$

$V25 = \{ A: A \rightarrow BC, B \in V22 , C \in V35\}$
$\quad U \{ A: A \rightarrow BC, B \in V23 , C \in V45\}$
$\quad U \{ A: A \rightarrow BC, B \in V24 , C \in V55\}$
$\quad = \Phi$

Step 5:

$V15 = \{ A: A \rightarrow BC, B \in V11 , C \in V25\}$
$\quad U \{ A: A \rightarrow BC, B \in V12 , C \in V35\}$
$\quad U \{ A: A \rightarrow BC, B \in V13 , C \in V45\}$
$\quad U \{ A: A \rightarrow BC, B \in V14 , C \in V55\}$
$\quad = \Phi$

Since V15 does not have S, therefore, $w \notin L(G)$

Page #172, prob#1, part3.

w = abbbb

Step1.

V11 = {A}
V22 = {B}
V33 = {B}
V44 = {B}
V55 = {B}

Step 2:

V12 = {A: A→BC, B∈ V11 , C∈ V22} = {S,B}
V23 = { A : A→BC, B∈ V22 , C∈ V33} = {A}
V34 = { A: A→BC, B∈ V33 , C∈ V44} = {A}
V45 = { A: A→BC, B∈ V44 , C∈ V55} = {A}

Step 3:

V13 = { A: A→BC, B∈ V11 , C∈ V23}
U { A: A→BC, B∈ V12 , ∈ V33}
= {A}

V24 = { A: A→BC, B∈ V22 , C∈ V34}
U { A: A→BC, B∈ V23 , C∈ V44}
= {S,B}

V35 = { A: A→BC, B∈ V33 , C∈ V45}
U { A: A→BC, B∈ V34 , C∈ V55}
= {S,B}

Step 4:

V14 = { A: A→BC, B∈ V11 , C∈ V24}
U { A: A→BC, B∈ V12 , C∈ V34}
U { A: A→BC, B∈ V13 , C∈ V44}

$= \{S,B\}$

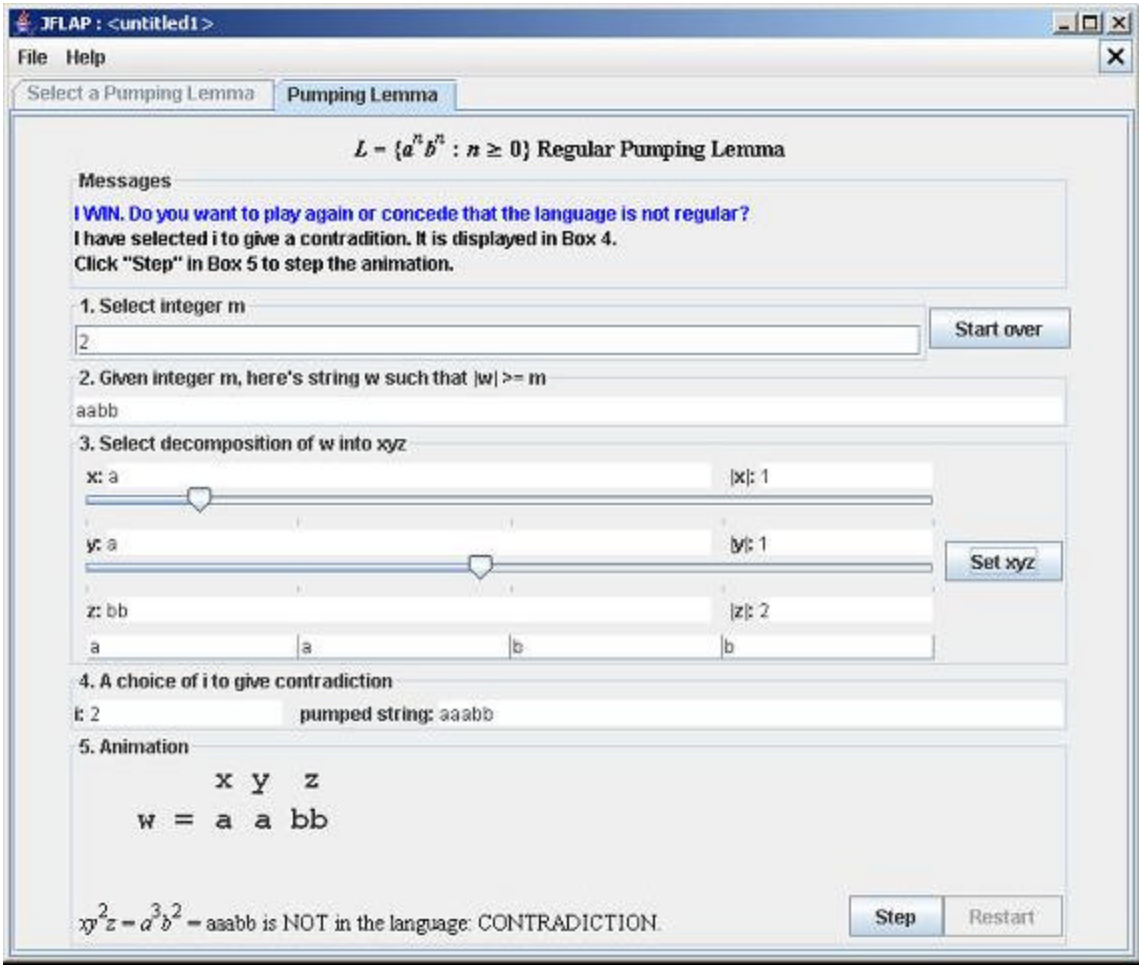
$V25 = \{ A: A \rightarrow BC, B \in V22 , C \in V35\}$
$\cup \{ A: A \rightarrow BC, B \in V23 , C \in V45\}$
$\cup \{ A: A \rightarrow BC, B \in V24 , C \in V55\}$
$= \{A\}$

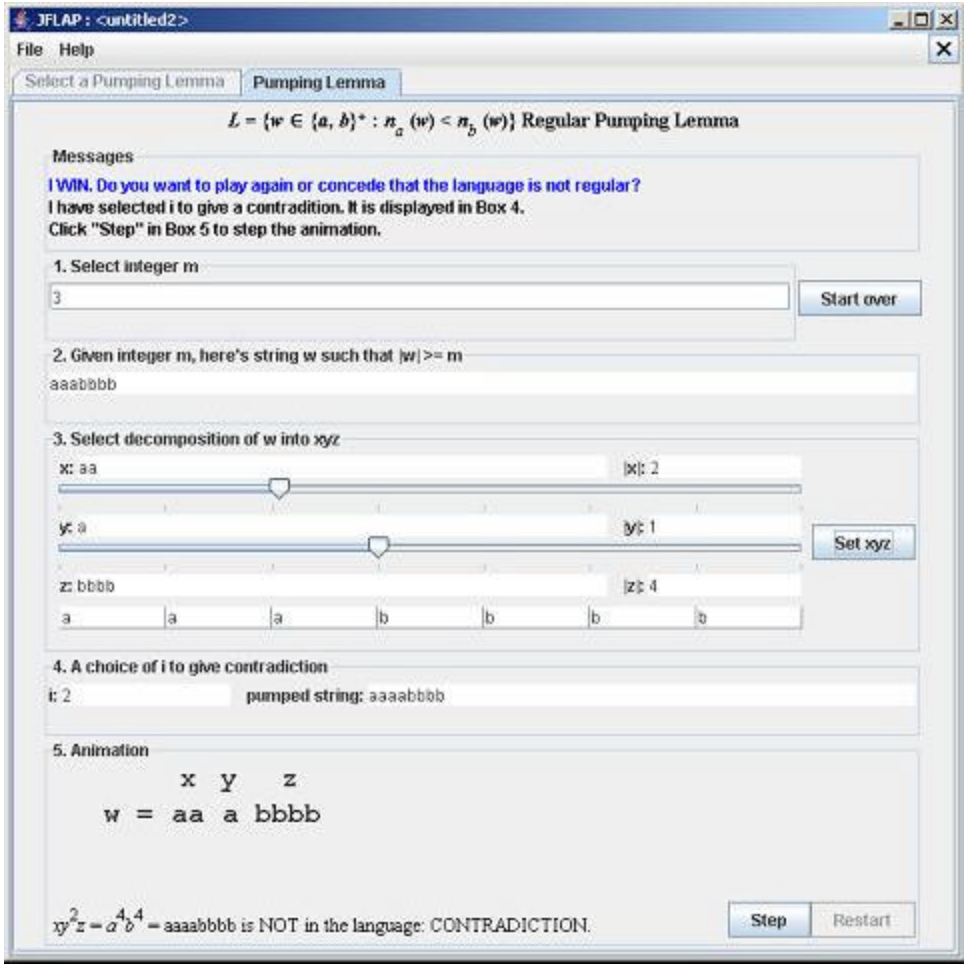
Step 5:

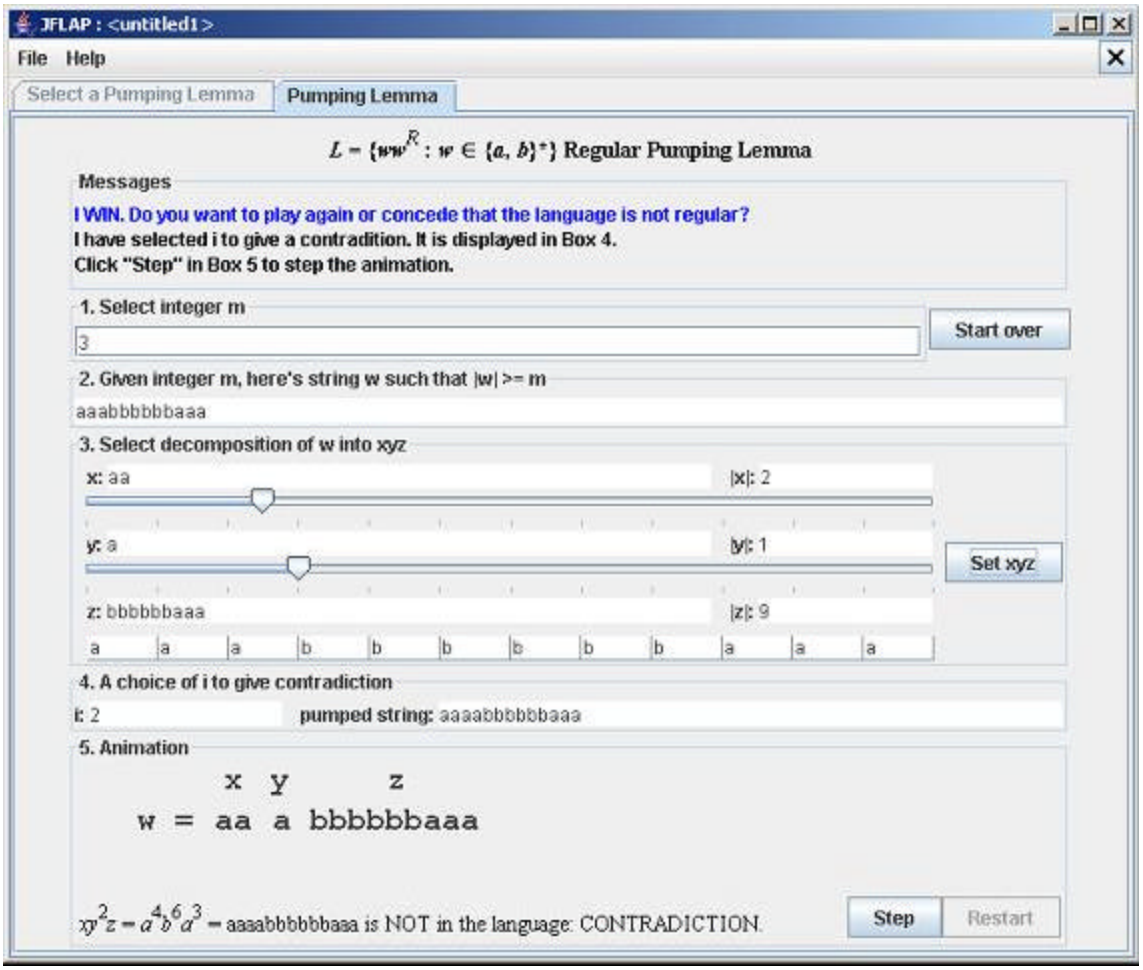
$V15 = \{ A: A \rightarrow BC, B \in V11 , C \in V25\}$
$\cup \{ A: A \rightarrow BC, B \in V12 , C \in V35\}$
$\cup \{ A: A \rightarrow BC, B \in V13 , C \in V45\}$
$\cup \{ A: A \rightarrow BC, B \in V14 , C \in V55\}$
$= \{A\}$

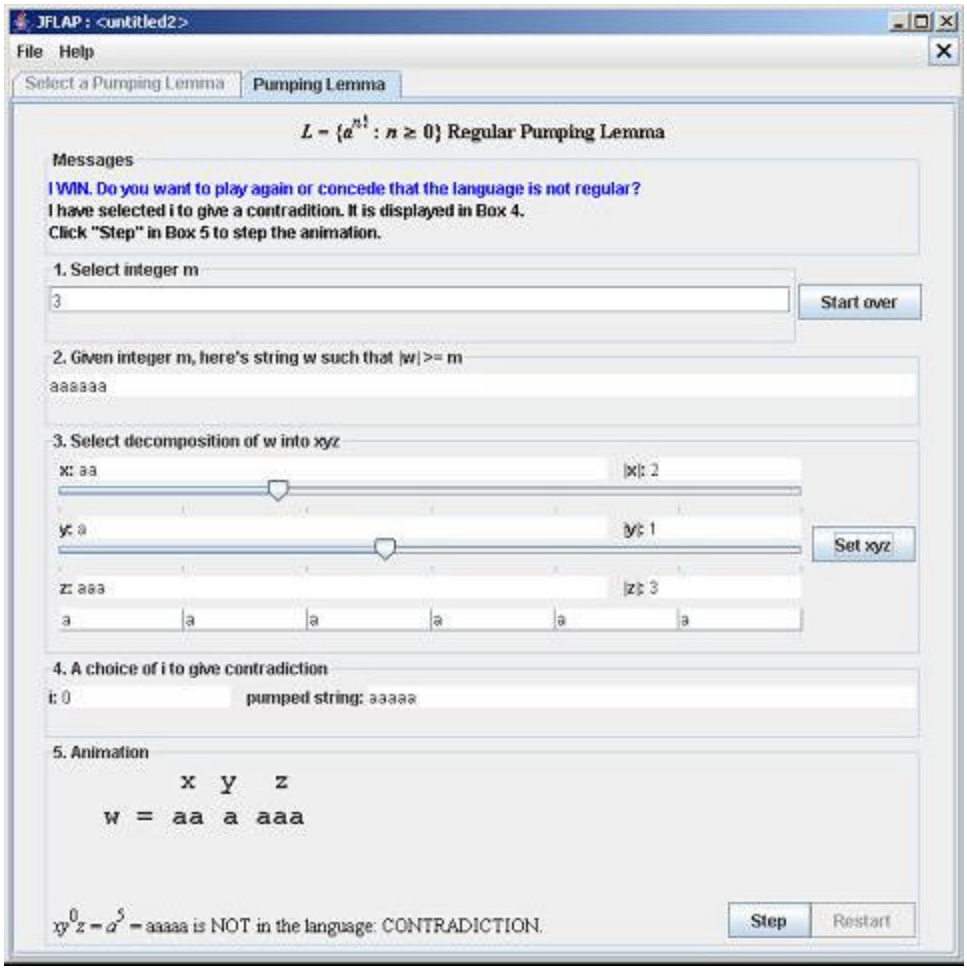
Since V15 does not have S, therefore, $w \notin L(G)$

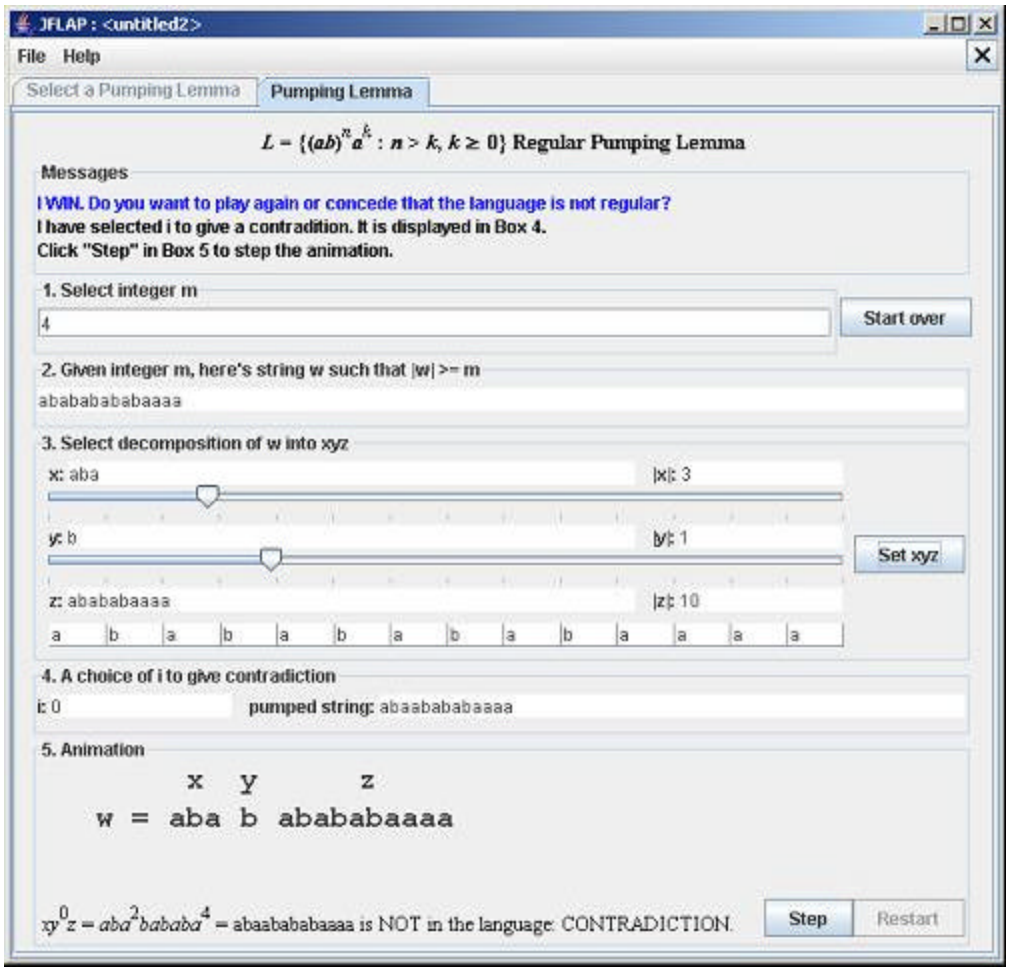
JFLAP Pumping Lemma Screen Shots:

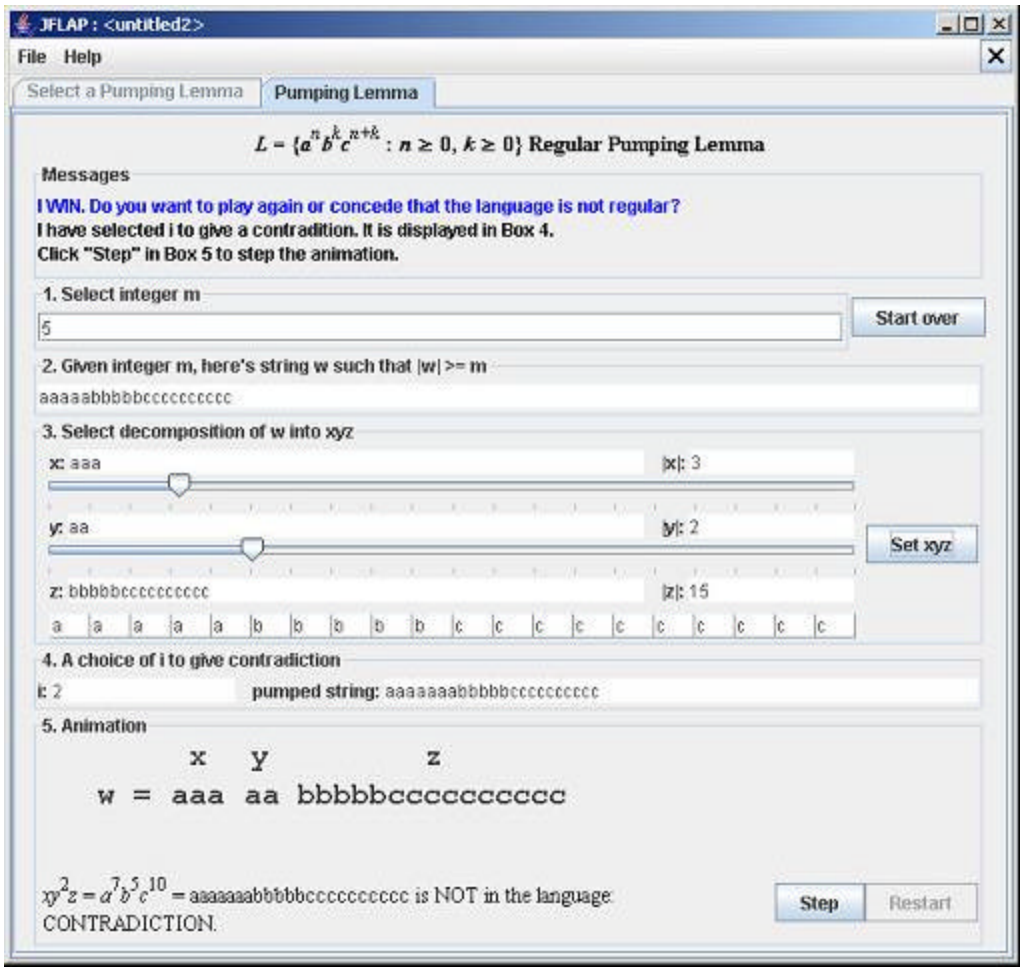


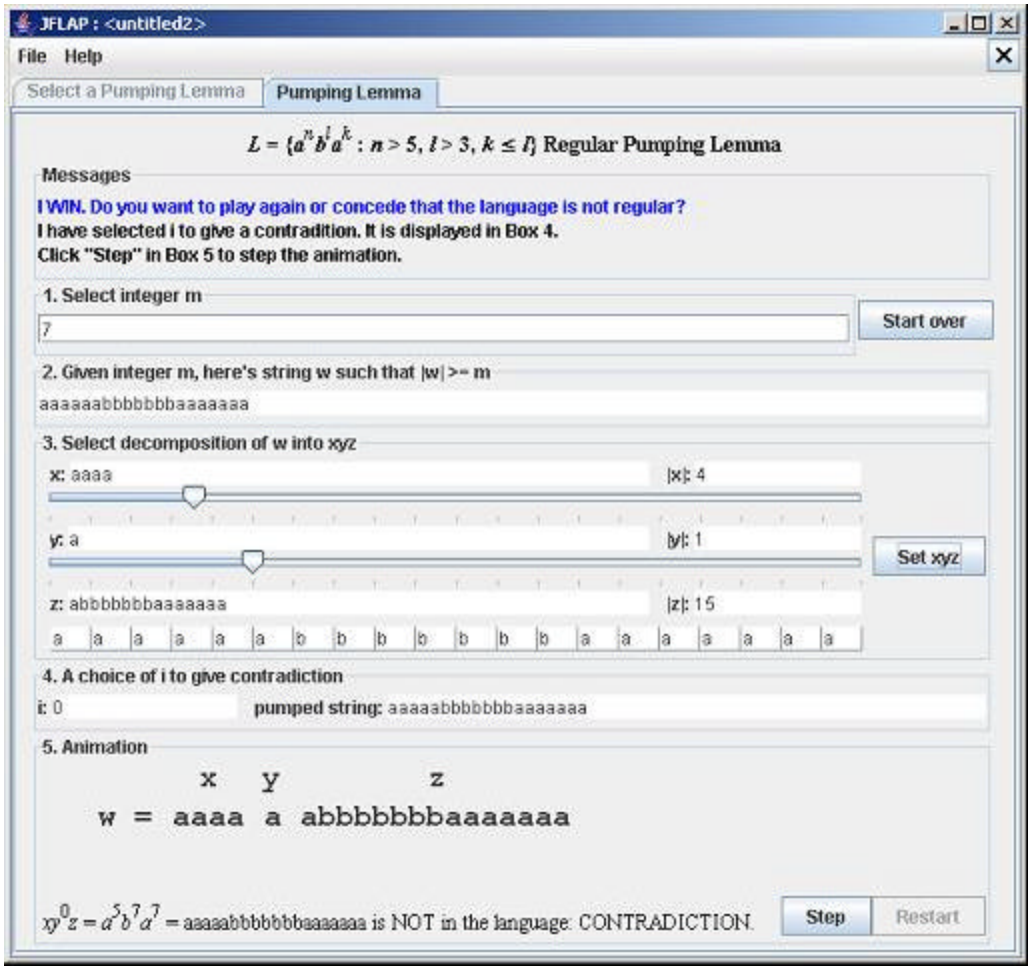

JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{w \in \{a, b\}^* : n_a(w) < n_b(w)\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
3
Start over
2. Given integer m, here's string w such that |w| >= m
aaabbbb
3. Select decomposition of w into xyz
x: aa
|x|: 2
y: a
|y|: 1
Set xyz
z: bbbb
|z|: 4
a a a b b b b
4. A choice of i to give contradiction
i: 2
pumped string: aaaabbbb
5. Animation
x y z
w = aa a bbbb
$xy^2z = a^4b^4$ = aaaabbbb is NOT in the language: CONTRADICTION.
Step
Restart

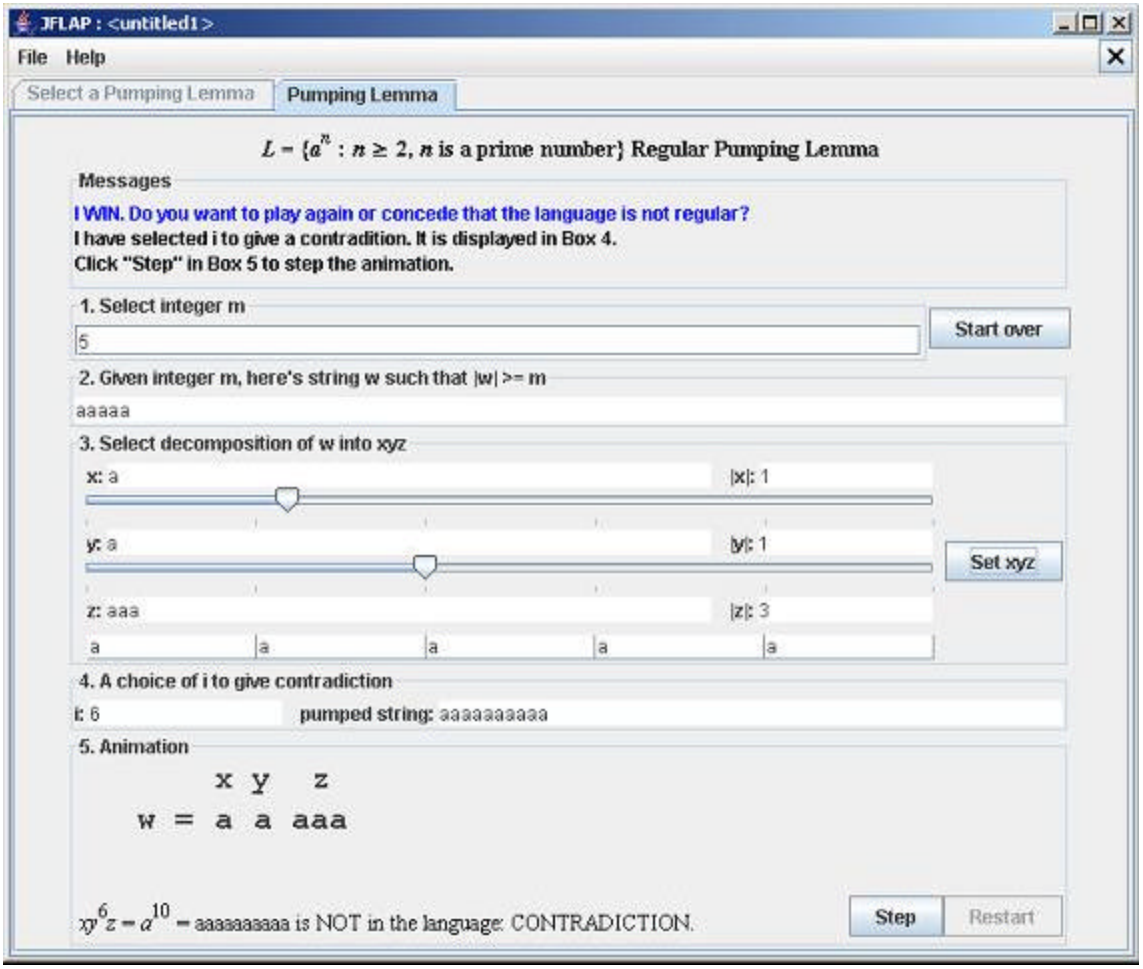

JFLAP : <untitled1>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
3
Start over
2. Given integer m, here's string w such that |w| >= m
aaabbbbbbaaa
3. Select decomposition of w into xyz
x: aa
|x|: 2
y: a
|y|: 1
Set xyz
z: bbbbbbaaa
|z|: 9
a a a b b b b b b a a a
4. A choice of i to give contradiction
i: 2
pumped string: aaaabbbbbbaaa
5. Animation
x y z
w = aa a bbbbbbaaa
$xy^2z = a^4b^6a^3$ = aaaabbbbbbaaa is NOT in the language: CONTRADICTION.
Step
Restart

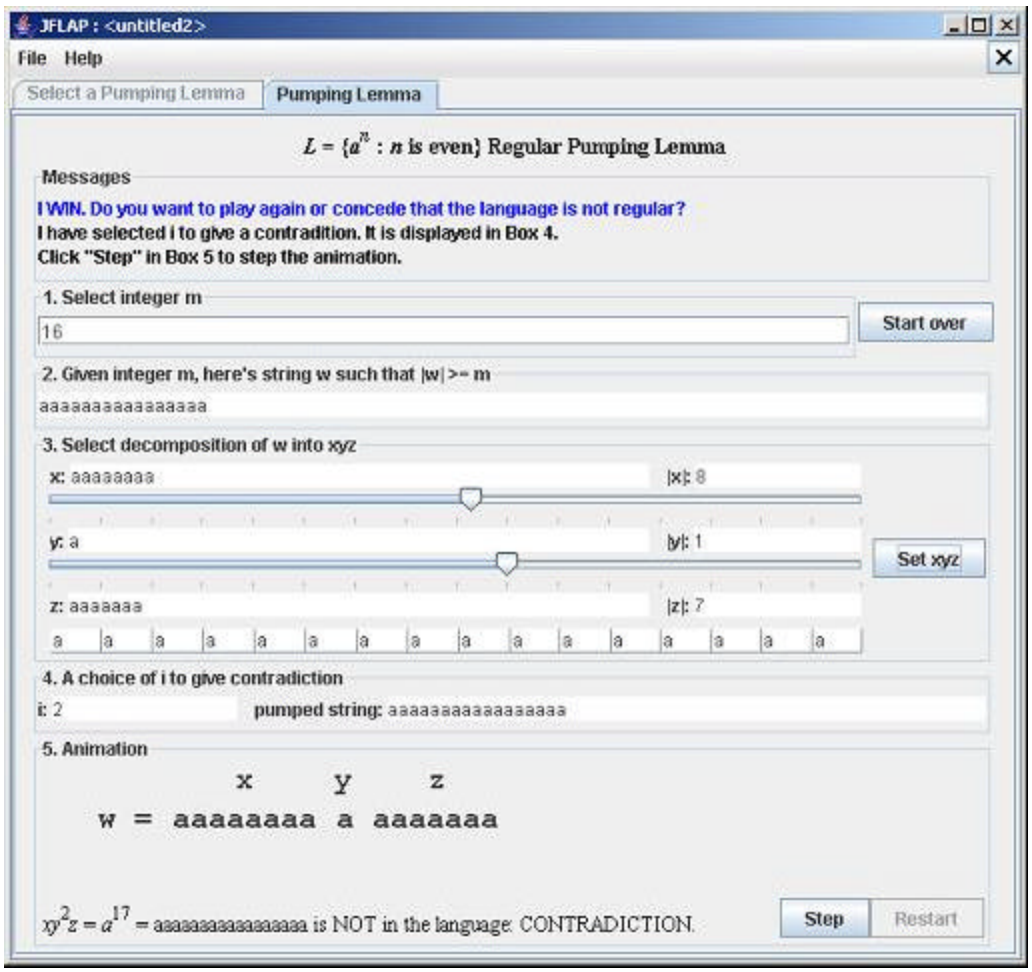

JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^{n!} : n \geq 0\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
3
Start over
2. Given integer m, here's string w such that |w| >= m
aaaaaa
3. Select decomposition of w into xyz
x: aa
|x|: 2
y: a
|y|: 1
Set xyz
z: aaa
|z|: 3
a a a a a a
4. A choice of i to give contradiction
i: 0
pumped string: aaaaa
5. Animation
x y z
w = aa a aaa
$xy^0z = a^5$ = aaaaa is NOT in the language: CONTRADICTION.
Step
Restart


JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{(ab)^n a^k : n > k, k \geq 0\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
4
Start over
2. Given integer m, here's string w such that |w| >= m
abababababaaaa
3. Select decomposition of w into xyz
x: aba
|x|: 3
y: b
|y|: 1
Set xyz
z: abababaaaa
|z|: 10
a b a b a b a b a b a a a a
4. A choice of i to give contradiction
i: 0
pumped string: abaababababaaaa
5. Animation
x y z
w = aba b abababaaaa
$xy^0z = aba^2bababa^4$ = abaababababaaaa is NOT in the language: CONTRADICTION.
Step
Restart


JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
5
Start over
2. Given integer m, here's string w such that |w| >= m
aaaaabbbbbccccccccc
3. Select decomposition of w into xyz
x: aaa
|x|: 3
y: aa
|y|: 2
Set xyz
z: bbbbbccccccccc
|z|: 15
a a a a a b b b b b c c c c c c c c c c c
4. A choice of i to give contradiction
i: 2
pumped string: aaaaaaabbbbbccccccccc
5. Animation
x y z
w = aaa aa bbbbbccccccccc
$xy^2z = a^7b^5c^{10}$ = aaaaaaabbbbbccccccccc is NOT in the language
CONTRADICTION.
Step
Restart


JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^n b^l a^k : n > 5, l > 3, k \leq l\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
7
Start over
2. Given integer m, here's string w such that |w| >= m
aaaaaabbbbbbbaaaaaaa
3. Select decomposition of w into xyz
x: aaaa
|x|: 4
y: a
|y|: 1
Set xyz
z: abbbbbbbaaaaaaa
|z|: 15
a a a a a a b b b b b b b a a a a a a a
4. A choice of i to give contradiction
i: 0
pumped string: aaaaabbbbbbbaaaaaaa
5. Animation
x y z
w = aaaa a abbbbbbbaaaaaaa
$xy^0z = a^5b^7a^7$ = aaaaabbbbbbbaaaaaaa is NOT in the language: CONTRADICTION.
Step
Restart


JFLAP : <untitled1>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^n : n \geq 2, n \text{ is a prime number}\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
5
Start over
2. Given integer m, here's string w such that |w| >= m
aaaaa
3. Select decomposition of w into xyz
x: a
|x|: 1
y: a
|y|: 1
Set xyz
z: aaa
|z|: 3
a a a a a
4. A choice of i to give contradiction
i: 6
pumped string: aaaaaaaaaa
5. Animation
x y z
w = a a aaa
$xy^6z = a^{10}$ = aaaaaaaaaa is NOT in the language: CONTRADICTION.
Step
Restart


JFLAP : <untitled2>
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{a^n : n \text{ is even}\}$ Regular Pumping Lemma
Messages
I WIN. Do you want to play again or concede that the language is not regular?
I have selected i to give a contradition. It is displayed in Box 4.
Click "Step" in Box 5 to step the animation.
1. Select integer m
16
Start over
2. Given integer m, here's string w such that |w| >= m
aaaaaaaaaaaaaaaa
3. Select decomposition of w into xyz
x: aaaaaaaa
|x|: 8
y: a
|y|: 1
Set xyz
z: aaaaaaa
|z|: 7
a a a a a a a a a a a a a a a a
4. A choice of i to give contradiction
i: 2
pumped string: aaaaaaaaaaaaaaaaa
5. Animation
x y z
w = aaaaaaaa a aaaaaaa
$xy^2z = a^{17}$ = aaaaaaaaaaaaaaaaa is NOT in the language: CONTRADICTION.
Step
Restart