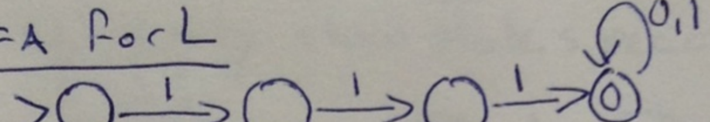
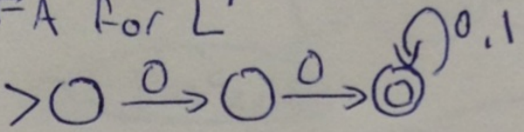
HW2 Grp1

(1) Let $L = \{\omega \mid \omega$ begins with three 1's$\}$
$L' = \{\omega \mid \omega$ begins with two 0's$\}$.
Draw NFAs for both languages. Then use the constructions from class to make NFAs for LL', $L \cup L'$ and L^*.

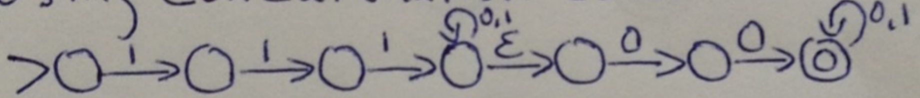
NFA for L



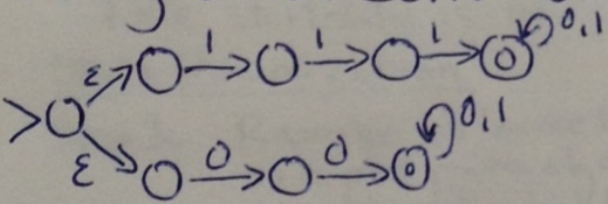
NFA for L'



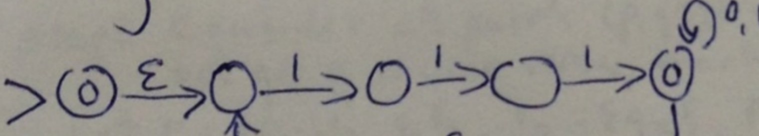
Using concatenation construction:



Using union construction:

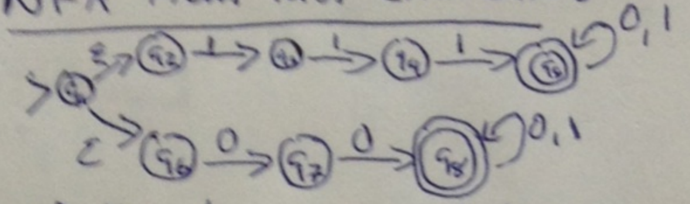


Using Kleene Star construction:



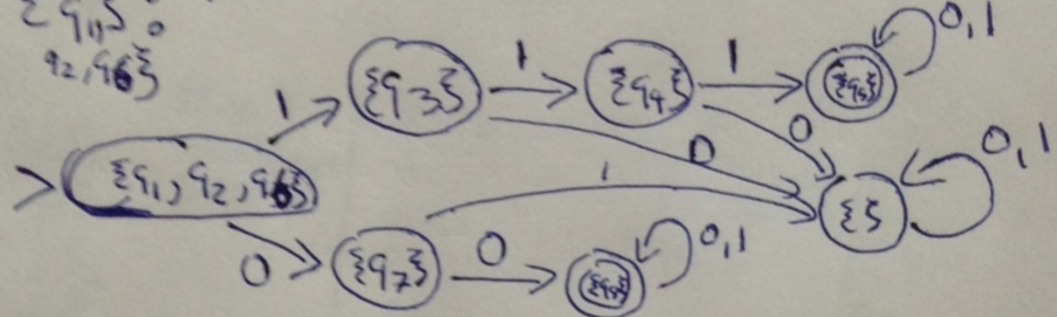
2 Use the power set construction to convert your NFA from the last exercise for $L \cup L'$ into a DFA then do minimization on the result.

NFA from last exercise:



Will only show states reachable from start state

$\{q_1\}$:
$q_2, q_6\}$



Note startstate is $E(\{q_1\}) = \{q_1, q_2, q_6\}$.

State Minimization

Step 1 Remove all inaccessible states using simple path algorithm.
we already implicitly did this above by not considering all states in power set.

step 2 Consider all pairs (p,q) of states. If $p \in F$ but $q \notin F$ or vice versa, mark (p,q) distinguishable

Let $r_1 = \{q_1, q_2, q_6\}$, $r_2 = \{q_3\}$, $r_3 = \{q_4\}$, $r_4 = \{q_5\}$
$r_5 = \{q_7\}$, $r_6 = \{q_8\}$, $r_7 = \{\}$, so have simpler names for states. Then get:

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1				X		X	
r_2				X		X	
r_3				X		X	
r_4	X	X	X		X		X
r_5				X		X	
r_6	X	X	X		X		X
r_7				X		X	

(prob 2 cont'd)

Step 3 Repeat until no previously unmarked pairs are marked

(a) For all (p, q) and all $a \in \Sigma$, compute $\delta(p, a) = p_a$ and $\delta(q, a) = q_a$. If (p_a, q_a) is marked as distinguishable, mark (p, q) as distinguishable

Pass 1 applying (a)

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1			X	X	X	X	
r_2			X	X	X	X	
r_3	X	X		X	X	X	X
r_4	X	X	X		X		X
r_5	X	X	X	X		X	X
r_6	X	X	X		X		X
r_7			X	X	X	X	

Example why (r_5, r_7) marked this round because $\delta(r_7, 0) = r_7$ (rejecting)

$\delta(r_5, 0) = r_6$ (accepting)

(r_6, r_7) marked previously

Pass 2 applying (a)

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1		X	X	X	X	X	
r_2	X		X	X	X	X	X
r_3	X	X		X	X	X	X
r_4	X	X	X		X		X
r_5	X	X	X	X		X	X
r_6	X	X	X		X		X
r_7		X	X	X	X	X	

Examply why (r_1, r_2) marked

$\delta(r_1, 1) = r_2$

$\delta(r_2, 1) = r_3$

(r_2, r_3) marked

Pass 3 applying (a)

	r_1	r_2	r_3	r_4	r_5	r_6	r_7
r_1		X	X	X	X	X	X
r_2	X		X	X	X	X	X
r_3	X	X		X	X	X	X
r_4	X	X	X		X		X
r_5	X	X	X	X		X	X
r_6	X	X	X		X		X
r_7	X	X	X	X	X	X	

Pass 4 no change so stop

Step 4 For each equivalence class create a state:

$[r_1] = \{r_1\}$

$[r_2] = \{r_2\}$

$[r_3] = \{r_3\}$

$[r_4] = \{r_4, r_6\}$) Same

$[r_5] = \{r_5\}$

$[r_6] = \{r_4, r_6\}$

$[r_7] = \{r_7\}$

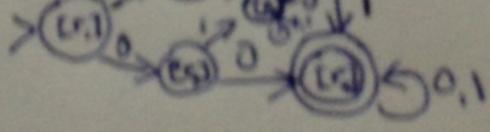
Step 5 For each rule $\delta(r, a) = s$, add transition $\delta([r], a) = [s]$

As an example, $\delta([r_1], 0) = [r_5]$.

Step 6 Set initial state to $[r_1]$

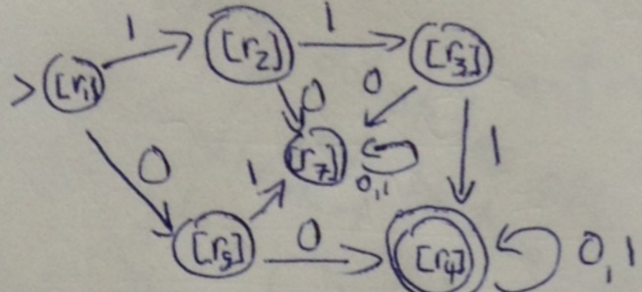
Step 7 Set final states $\{[r_4]\}$

Get Machine:

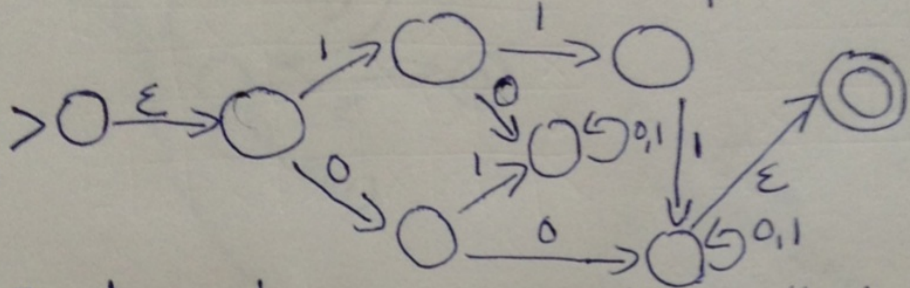


③ Use the construction from class to convert your DFA to a regular expression.

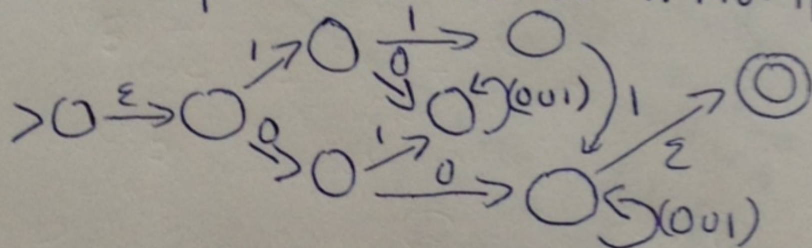
<u>DFA from last exercise</u>



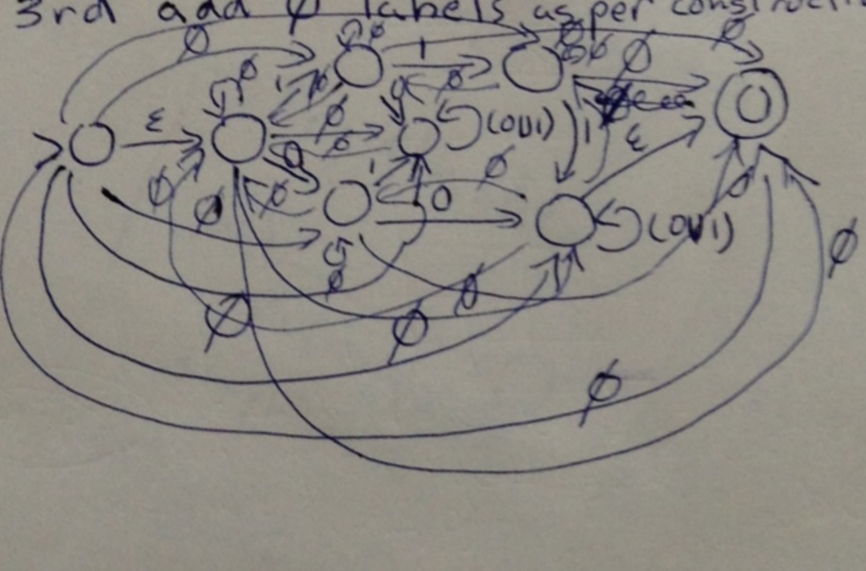
1st add new start and accept states



2nd replace arrows with multiple labels with union



3rd add ∅ labels as per construction

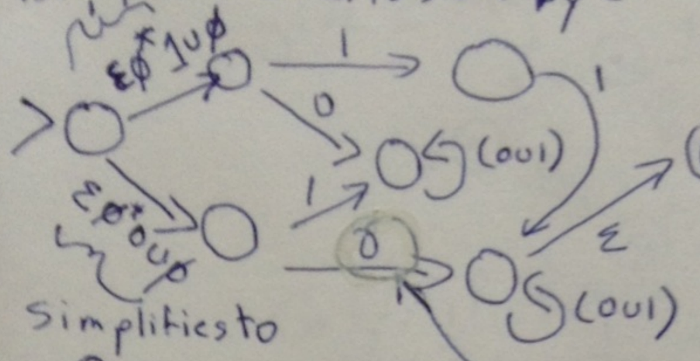


← very messy will suppress ∅ states on next page so can read diagrams

3 cont'd

Start Ripping out states other than first and last till get to two states

1st rip state connect to start by ε.

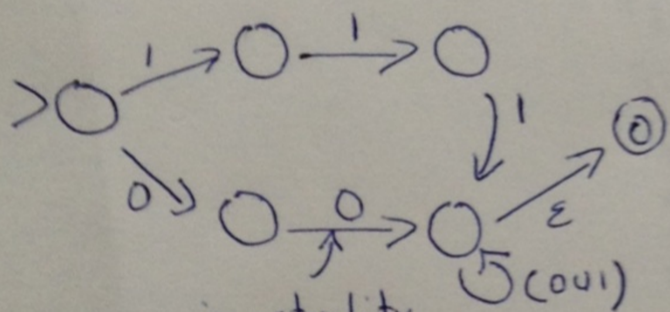


simplifies to 1

simplifies to 0

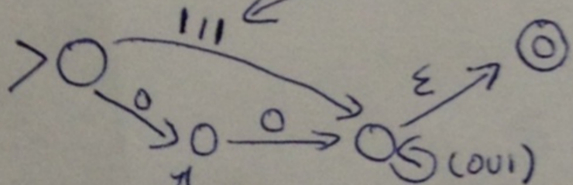
recall am suppressing ∅ transitions for clarity. Also, transitions like , would have been updated too. For example, to $0 \cup \emptyset\emptyset^*\emptyset$ which simplifies to 0

2nd rip trapstate

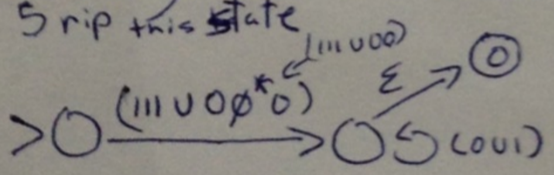


in actuality this would have been updated to: $0 \cup 1(0 \cup 1)^*\emptyset$ which simplifies to 0

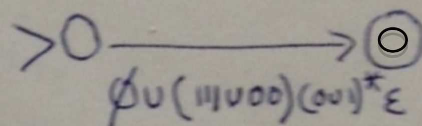
3, 4 rip top two 1 transition states after simplification



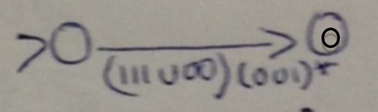
5 rip this state



Rip middle state

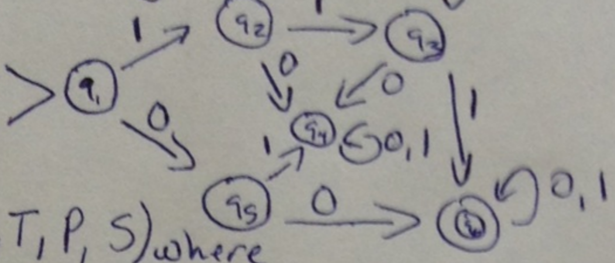


simplifies to



final regex

4 Using the construction from class give a right-linear grammar for the DFA:



$G = (V, T, P, S)$ where

$V = \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$T = \{0, 1\}$

$P = \{$ $q_1 \to 1q_2,$
$q_1 \to 0q_5,$
$q_2 \to 1q_3,$
$q_2 \to 0q_4,$
$q_3 \to 1q_6,$
$q_3 \to 0q_4,$
$q_4 \to 1q_4,$
$q_4 \to 0q_4,$
$q_5 \to 1q_4$
$q_6 \to 0q_6$
$q_6 \to 1q_6$
$q_6 \to \varepsilon$ $\}$

$S = q_1$